

## **EXPRESSIVE POWER OF THE SCHEMATIC PROTECTION MODEL\***

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### **Abstract**

In this paper we show that the Schematic Protection Model (SPM) subsumes several well-known protection models as particular instances. We show this for a diverse collection of models including the Bell-LaPadula multilevel security model, take-grant models, and grammatical protection systems. Remarkably SPM subsumes these models within its known efficiently decidable cases for safety analysis (i.e., the determination or whether or not a given privilege can possibly be acquired by a particular subject). Therefore SPM subsumes these models not only in terms of its expressive power but also in terms of safety analysis. This is in sharp contrast to the Harrison-Ruzzo-Ullman (HRU) access-matrix model. HRU does subsume all the models discussed in this paper in terms of expressive power. However, all known constructions of these models in HRU require multi-conditional commands (i.e., commands whose conditions have two or more terms), whereas safety is undecidable in HRU even for bi-conditional commands (i.e., commands whose conditions have exactly two terms).

### **1 Introduction**

Access controls for protection and sharing of information and physical resources are an essential component of any multi-user computer system. For this purpose, these systems are typically viewed as consisting of subjects and objects. Subjects are generally the active entities such as users or processes, while objects are passive entities such as text files. Protection is enforced by ensuring that subjects can execute only those operations for which they are authorized.

Access controls are useful to the extent they meet the user community's needs. They need to be flexible so that individual users can specify access of other users

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to the objects they control. At the same time the discretionary power given to individual users must be constrained to meet the overall objectives and policies of an organization. For example, members of a project team might be allowed to freely share project documents with each other but only the project leader is authorized to allow non-members to read project documents.

The *protection state* of a system is defined by the *privileges\** in subjects' *domains* at a given moment. Hereafter, we understand state to mean protection state. *Inert privileges* authorize operations that do not modify the state, e.g., reading or writing a file. *Control privileges* authorize control operations that modify the protection state, e.g., user X authorizes user Y to read file Z. Control privileges define the dynamics of authorization. Once the initial state has been established,<sup>†</sup> the protection state evolves by the autonomous actions of subjects constrained by control privileges. The challenge is to ensure that all reachable states conform with the policy that the security administrator wishes to implement.

A protection model provides a framework for specifying the dynamics of the protection state. This is usually done by stating rules which prescribe the authorization for making incremental changes in the state. We call such a collection of rules an *authorization scheme*, often abbreviated simply as *scheme*. To understand the implications of a scheme it must be possible to determine the cumulative effect of authorized incremental changes in the protection state. The incremental state changes authorized by a scheme may appear innocent enough in isolation, although their cumulative effect turns out to be undesirable. So for a given initial state and authorization scheme, we need to characterize protection states that are reachable.

This problem was first identified in [14] where it is called the *safety problem*. In its most basic form, the safety question asks: is there a reachable state in which a particular subject possesses a particular privilege which it did not previously possess? It is the fundamental question which a protection model must confront. Since subjects are usually authorized to create new subjects and objects, the system is unbounded and it is not certain that such analysis will be decidable, let alone tractable, without sacrificing generality.

Safety analysis becomes particularly complex when control privileges can themselves be dynamically acquired. To illustrate the need for propagating control privileges, consider the example above where only the project leader is authorized to allow non-members to read project documents. This policy can be enforced by giving the project leader a special control privilege not available to ordinary project members. Suppose in addition we wish to allow the project leader to delegate this special authority to a project member, say while the project leader is absent on a business trip. This can be achieved by allowing the project leader to temporarily grant the special control privilege to project members.

Analysis issues were first formalized by Harrison, Ruzzo and Ullman [14] in the

\*We view a privilege as an undefined primitive concept. For the most part, privileges can be treated as synonymous to access rights. However, there are privileges such as security level and type which are usually represented as attributes of subjects and objects rather than as access rights.

†The initial state is established by the security administrator at the moment of system generation. The mechanics of this procedure are inherently implementation dependent, and therefore not modeled within SPM.

context of the well-known access matrix each subject and a column for each subject contains symbols called rights which authorize entity Y. An authorization scheme is defined to have a condition part and a body. The condition part must exist in the matrix before the body part. The body consists of a sequence of primitive operations: enter or delete a right from a cell of the matrix, destroy an existing row or column. The

In the general HRU setting safety is undecidable even if the condition part has only one term and there are no delete or destroy operations. Assumptions from which undecidability does not appear to be any natural and useful restriction is efficiently decidable [14, 15]. Specifically, the condition part has only one term) but is undecidable for mono-conditional monotonic commands [15] (i.e., commands whose condition part has two or more terms). Practical systems require multi-conditional condition part has two or more terms).

The schematic protection model (SPM) handles this situation. SPM provides considerably more flexibility in subjects and objects into protection types. The protection state in SPM consists of tickets (capabilities). An authorization scheme is specified in terms of protection types. A subject is authorized by a can-create binary relation. This relation is acyclic, and in certain cases it is also transitive. On the other hand with arbitrary cycles. Fortunately, it appears that SPM schemes are decidable under ability constraints, as demonstrated by several examples of [32, 33, 34, 36].

Our objective in this paper is to demonstrate that known protection models as special cases of the LaPadula multilevel security model [3] and the schematic protection systems [10, 22] are

Remarkably in all our constructions the safety analysis. Therefore SPM subsumes the safety analysis power but also in terms of safety analysis. SPM can simulate all the models discussed in this paper. Weak safety properties HRU is unable to handle. Decidable classes for safety.<sup>‡</sup> However, HRU require multi-conditional commands.

‡This statement is true if we require that the condition part of the "valent" HRU system. For weaker notions of equivalence, the "valent" HRU system for any model that has a safety property is Section 4.

at the same time the discretionary power given to meet the overall objectives and policies members of a project team might be allowed to access each other but only the project leader is allowed to read project documents.

Control is exercised by the *privileges\** in subjects' *domains* to understand state to mean protection state. Control operations that do not modify the state, e.g., reading files, authorize control operations that modify the state, e.g., user Y to read file Z. Control privileges are defined since the initial state has been established,<sup>†</sup> and autonomous actions of subjects constrained by control to ensure that all reachable states conform with the project leader's wishes to implement.

A framework for specifying the dynamics of the system is given by stating rules which prescribe the authorized actions in the state. We call such a collection of rules a *scheme*. To understand the system is possible to determine the cumulative effect of the control on the protection state. The incremental state transition is near innocent enough in isolation, although the overall effect is undesirable. So for a given initial state and control rules, we characterize protection states that are reachable. This is the *safety problem* [14] where it is called the *safety problem*. In particular, the question asks: is there a reachable state in which a subject has a particular privilege which it did not previously have? This is the problem which a protection model must confront. To understand the system, to create new subjects and objects, the system must be able to perform such analysis will be decidable, let alone

is complex when control privileges can themselves be controlled. To illustrate the need for propagating control rules, we consider a policy where only the project leader is authorized to read project documents. This policy can be enforced by a control rule which is a privilege not available to ordinary project members. To allow the project leader to delegate this control to other members while the project leader is absent on a project, we consider allowing the project leader to temporarily delegate control to other project members.

Control is exercised by Harrison, Ruzzo and Ullman [14] in the context of the well-known access matrix model [13, 17]. The matrix has a row for each subject and a column for each subject or object. The [X,Y] cell of the matrix contains symbols called rights which authorize subject X to perform operations on entity Y. An authorization scheme is defined by a set of commands. Each command has a condition part and a body. The condition specifies the rights that are required to exist in the matrix before the body can be executed for its actual arguments. The body consists of a sequence of primitive operations. The primitive operations enter or delete a right from a cell of the matrix, create a new row or column, or destroy an existing row or column. This model is hereafter called HRU.

In the general HRU setting safety is undecidable [14]. Furthermore safety remains undecidable even if the condition part of each command has at most two terms and there are no delete or destroy operations in the body [15]. The very weak assumptions from which undecidability follows are most disappointing. There does not appear to be any natural and useful special case of this model for which safety is efficiently decidable [14, 15]. Specifically, safety in HRU is known to be decidable for mono-conditional monotonic commands [15] (i.e., commands whose condition part has only one term) but is undecidable even for bi-conditional monotonic commands [15] (i.e., commands whose condition part has exactly two terms). Most practical systems require multi-conditional commands (i.e., commands whose condition part has two or more terms).

The schematic protection model (SPM) [34] was developed in response to this situation. SPM provides considerably more structure than HRU. It classifies subjects and objects into protection types. The dynamic component of a protection state in SPM consists of tickets (capabilities). The key idea is that the authorization scheme is specified in terms of protection types. In particular, subject creation is authorized by a can-create binary relation on types. Safety is decidable provided this relation is acyclic, and in certain cases even if it has cycles of length one [34]. On the other hand with arbitrary cycles in can-create, safety is undecidable [38]. Fortunately, it appears that SPM schemes of practical interest satisfy the decidability constraints, as demonstrated by the constructions of this paper and the examples of [32, 33, 34, 36].

Our objective in this paper is to demonstrate that SPM subsumes several well-known protection models as special cases. Specifically, we show that the Bell-LaPadula multilevel security model [3], take-grant models [16, 21, 41] and grammatical protection systems [10, 22] are particular instances of SPM.

Remarkably in all our constructions the resulting SPM scheme has efficient safety analysis. Therefore SPM subsumes these models not only in terms of its expressive power but also in terms of safety analysis. This is in sharp contrast to HRU. HRU can simulate all the models discussed in this paper. However, because of its very weak safety properties HRU is unable to subsume these models within its known decidable classes for safety.<sup>‡</sup> However, all known constructions of these models in HRU require multi-conditional commands (i.e., commands whose conditions have

† This statement is true if we require that each model be simulated by a "behaviorally equivalent" HRU system. For weaker notions of equivalence it is always possible to construct an "equivalent" HRU system for any model that has a decidable safety problem. This issue is discussed in Section 4.

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two or more terms), whereas safety is undecidable in HRU even for bi-conditional commands (i.e., commands whose conditions have exactly two terms) [15]. In SPM, on the other hand, these models are all simulated by acyclic attenuating schemes which are known to have decidable safety [34].

Taken collectively our constructions demonstrate that the safety results for SPM subsume a diversity of published safety results for protection models. In and of itself each construction is also notable for the following reasons.

- The construction for multilevel security models shows that the traditional black-and-white distinction between mandatory and discretionary controls in the Bell-LaPadula model has an alternate expression in SPM in terms of constraints on the propagation of access rights. The SPM viewpoint has the advantage of providing explicit machinery for formulating policies "in between" these two extremes.
- The construction for theft in take-grant models shows a great advantage of SPM whereby assumptions about the behavior of subjects are easily specified as part of a scheme. SPM, therefore, gives us a powerful framework for investigating the consequences of assumptions about trusted behavior.
- Finally, the construction for grammatical systems demonstrates the ability of SPM to simulate models whose control operations are at first sight quite contrary to SPM control operations. It also completes the simulation of take-grant analysis within SPM, since aspects of this analysis require a combination of our constructions for theft and grammatical systems.

The rest of the paper is organized as follows. We begin by reviewing SPM and its analysis results respectively in Sections 2 and 3. Section 4 discusses the concept of equivalence among systems and subsumption among models. Section 5 shows that the Bell-LaPadula multilevel security model [3] and several variations of it are expressible as SPM schemes. Section 6 considers how several variations of the take-grant model [16, 21, 41] can be simulated in SPM. Section 7 shows that grammatical protection systems [10, 22] are particular instances of SPM. The constructions of Sections 5, 6 and 7 are largely independent of each other and can be read in any order. Section 8 concludes the paper.

## 2 The Schematic Protection Model

In this Section we review the definition of SPM. Our review is necessarily brief. Motivational details for various components of the model are given in [34]. The only difference in notation with respect to [34] is in describing the create-rules.

SPM recognizes two kinds of *entities*, called *subjects* and *objects*, in a system. Subjects can possess privileges and may or may not be active agents in the system. Objects, on the other hand, are purely containers of information. They do not possess privileges and are inherently passive. SPM regards subjects and objects as mutually exclusive. In the literature, subjects are often defined to be a subset of objects. This amounts to calling what SPM calls entities as objects and coining some other term (say, pure objects) for entities which are not subjects. In SPM

terminology a process is a subject, and on subjects. In the literature, a process about the process executing operations on an object when we talk about operations on an object of terminology are quite straightforward while reading the paper.

The key notion in SPM is that all instances of the same protection type are created together. Hereafter, we understand *type* to mean a protection type. In the typing in that every entity is created together with its protection type thereafter. The domain of an SPM subject is defined by the *scheme* and a dynamic set of objects. The scheme is defined in terms of types and objects. The scheme is first set-up and thereafter does not change. The details of the scheme while details are reflected in the objects.

Tickets are privileges of the form  $Y/u$  where  $Y$  is some unique entity and the right symbol  $u$  is a set of rights to perform some operation(s) on  $Y$ . Tickets are created at will by a subject. They can be acquired by a subject which comprise the scheme. The assumption that a right symbol simplifies the formal framework. Multiple right symbols then correspond to multiple tickets in this manner, e.g.,  $Y/uvw$  d

### 2.1 Types And Right Symbols

The first step in defining a scheme is to define the set of object types TO and subject types TS. Their union identifies classes of entities which have a certain type. For subjects this may be membership in a group, such as project leader, or authority in a group, such as project leader. For objects such as an internal document or a public document. Lower case italics and entities in upper case roman script are used to name sets, functions, and types and entities respectively. The type of an entity is denoted by  $t$ .

The next (or perhaps concurrent) step is to define the set of rights by tickets. For this purpose the set of rights is divided into disjoint subsets: the inert rights RI and the active rights. The active rights are the typical read, write, execute, etc. The interpretation of control rights will be given later. SPM does not interpret the inert rights, but the security administrator is free to define the interpretation for the particular system of interest. For example, that  $r$  means read and  $w$  means write, etc. The scheme. On the other hand, the interpretation of the SPM scheme in terms of link prediction.

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terminology a process is a subject, and operations (e.g., signal) can be performed  
 on subjects. In the literature, a process is often called a subject when we talk  
 about the process executing operations on other objects, whereas it is called an  
 object when we talk about operations performed on that process. These nuances  
 of terminology are quite straightforward, but it is important to keep them in mind  
 while reading the paper.

The key notion in SPM is that all entities are instances of protection types.  
 Instances of the same protection type are treated uniformly by control privileges.  
 Hereafter, we understand *type* to mean protection type. SPM assumes strong  
 typing in that every entity is created to be of a specific type which cannot change  
 thereafter. The domain of an SPM subject has two parts: a static type-dependent  
 part defined by the *scheme* and a dynamic part consisting of *tickets* (capabilities).  
 The scheme is defined in terms of types by the security administrator when a system  
 is first set-up and thereafter does not change. Major policy decisions are built into  
 the scheme while details are reflected in the initial distribution of tickets.

Tickets are privileges of the form  $Y/x$ .  $Y/x$  is an ordered pair where  $Y$  identifies  
 some unique entity and the right symbol  $x$  authorizes the possessor of this ticket to  
 perform some operation(s) on  $Y$ . Tickets are unforgeable and cannot be generated  
 at will by a subject. They can be acquired only in accordance with specific rules  
 which comprise the scheme. The assumption that a ticket carries only one right  
 symbol simplifies the formal framework without loss of generality. Capabilities with  
 multiple right symbols then correspond to sets of tickets. We often abbreviate sets  
 of tickets in this manner, e.g.,  $Y/uvw$  denotes the set of tickets  $\{Y/u, Y/v, Y/w\}$ .

### 2.1 Types And Right Symbols

The first step in defining a scheme is to specify the disjoint sets of object types  
 TO and subject types TS. Their union T is the entire set of entity types. Types  
 identify classes of entities which have common properties for security purposes.  
 For subjects this may be membership in a department or a particular position of  
 authority in a group, such as project leader. For objects this may be a classification  
 such as an internal document or a public document. Types are usually named in  
 lower case italics and entities in upper case roman script. Similarly italics and  
 roman script are used to name sets, functions and relations whose domains involve  
 types and entities respectively. The type of entity  $Y$  is denoted by *type*( $Y$ ).

The next (or perhaps concurrent) step is to define the right symbols carried  
 by tickets. For this purpose the set of right symbols R is partitioned into two  
 disjoint subsets: the inert rights RI and the control rights RC. Examples of inert  
 rights are the typical read, write, execute and append access rights for a file. The  
 interpretation of control rights will be discussed shortly. We emphasize that SPM  
 does not interpret the inert rights, but rather treats them as abstract symbols. The  
 security administrator is free to define RI as the collection of symbols appropriate  
 for the particular system of interest. The interpretation of these symbols, e.g.,  
 that  $r$  means read and  $w$  means write, is informal and is not specified in the SPM  
 scheme. On the other hand, the interpretation of the control rights is specified in  
 the SPM scheme in terms of link predicates and filter functions.

Every right symbol  $x$  comes in two variations  $x$  and  $xc$ , where  $c$  is the copy

flag. The only difference between  $Y/x$  and  $Y/xc$  is that the former ticket cannot be copied from one subject to another whereas the latter may be, provided certain additional conditions to be defined shortly are true. It follows that presence of  $Y/xc$  in a domain implies presence of  $Y/x$  but not vice versa. We use  $x:c$  to denote  $x$  or  $xc$  with the understanding that multiple occurrences of  $x:c$  in the same context are either all read as  $x$  or all as  $xc$ . When used with multiple right symbols on a ticket the copy flag applies to each symbol, that is  $Y/uvwc$  denotes  $\{Y/uc, Y/vc, Y/wc\}$ .

We denote the type of a ticket  $Y/x:c$  by  $type(Y/x:c)$  and define it to be the ordered pair  $type(Y)/x:c$ . That is, the type of a ticket is determined by the type of entity it addresses and the right symbol it carries. Conventions for representing tickets, especially regarding the copy flag, extend in an obvious way to ticket types. In particular  $type(Y/x)$  and  $type(Y/xc)$  are different. This is an important distinction because of the role of the copy flag. The entire set of ticket types is  $T \times R$ .

The remaining components of a scheme are defined in terms of functions and relations involving the sets  $TS$ ,  $T$  and  $T \times R$ . SPM requires that  $T$  and  $R$  be finite, so a scheme is defined by finite sets, relations and functions. SPM recognizes two operations that change the protection state: copy and create.<sup>§</sup>

## 2.2 The Copy Operation

The copy operation moves a copy of a ticket from the domain of one subject to the domain of another, leaving the original ticket intact. We often speak of copying a ticket from one subject to another although technically a ticket is copied from one subject's domain to another's domain.

The copy operation requires three independent pieces of authorization. A formal statement of these three conditions is given in Section 2.2.2.

1. The original ticket in the source subject's domain must carry the copy flag.
2. There must be a link from the source subject to the destination subject. In general an SPM scheme defines a collection of link predicates  $\{\text{link}_i\}$  for this purpose. A  $\text{link}_i$  is said to exist from one subject to another provided the predicate  $\text{link}_i$  evaluates to true in a given state.
3. Finally the filter function  $f_i$  associated with the link predicate  $\text{link}_i$  must also authorize the operation.

The subscript  $i$  is used to distinguish one link predicate from another as well as to maintain the association between link predicates and filter functions. The symbol  $i$  is usually chosen to have some mnemonic significance with respect to the control rights which establish the link or the purpose of the link. We now formally define links and filter functions.

<sup>§</sup>In its original formulation SPM included a third operation called demand. Demand is not used in the constructions of this paper and is known to be formally redundant [37].

### 2.2.1 Link Predicates

In an SPM scheme a finite collection takes two subjects, say  $U$  and  $V$ , as arguments. If a link predicate is true, it establishes a connection from  $U$  to  $V$ . The definition of each link predicate is a combination of control tickets for  $U$  and  $V$ . The definition of a link predicate is that link predicates can therefore be defined for two subjects of concern and that too only for these two subjects. That the definition of a link predicate is not the absence of tickets is a well-known fact. In this case we also allow a link predicate which is true if either of two link predicates have the following definition.

**Definition 1** Let  $dom(U)$  be the set of subjects in the domain of  $U$ . A link predicate  $\text{link}_i(U, V)$  with  $U$  and  $V$  in the domain of  $U$  and  $V$  is a conjunction or disjunction, but not negation, of the form  $U/z \in dom(U)$ ,  $U/z \in dom(V)$ ,  $V/z \in dom(V)$ .

For a given state if  $\text{link}_i(A, B)$  is true we emphasize the existence of a link is needed from  $A$  to  $B$ . Examples of local link pre

1.  $\text{link}_{tg}(U, V) \equiv U/t \in dom(V) \vee V/t \in dom(U)$
2.  $\text{link}_t(U, V) \equiv U/t \in dom(V)$
3.  $\text{link}_g(U, V) \equiv V/g \in dom(U)$
4.  $\text{link}_{sr}(U, V) \equiv U/r \in dom(V) \wedge V/r \in dom(U)$
5.  $\text{link}_b(U, V) \equiv U/b \in dom(U)$
6.  $\text{link}_p(U, V) \equiv V/p \in dom(V)$
7.  $\text{link}_{bp}(U, V) \equiv U/b \in dom(U) \wedge V/p \in dom(V)$
8.  $\text{link}_q(U, V) \equiv \text{true}$

The first example is from the take-grant mechanism where rights are respectively read as take and grant. The second and third just one of these privileges [23]. The fourth is from the receive mechanism [28, 32] where the  $s$  and  $r$  stand for send and receive. The first four cases are defined for a subject in the domain or vice versa. The next three are defined in terms of a control ticket for  $U$  in  $U$ 's domain. The last one is unique in that it requires no tickets for a link to exist. The possibilities for defining link predicates are infinite. The kind defined above will suffice in practice, even for arbitrarily complex ones.

$Y/xc$  is that the former ticket cannot be true as the latter may be, provided certain conditions are true. It follows that presence of  $Y/xc$  does not imply vice versa. We use  $x:c$  to denote occurrences of  $x:c$  in the same context. We use  $x:c$  with multiple right symbols on a ticket, that is  $Y/uvw$  denotes  $\{Y/uc, Y/vc,$

$type(Y/x:c)$  and define it to be the type of a ticket is determined by the type of control it carries. Conventions for representing tickets, extend in an obvious way to ticket types. Ticket types are different. This is an important property of a ticket. The entire set of ticket types is

are defined in terms of functions and control rights. SPM requires that  $T$  and  $R$  be finite, sets and functions. SPM recognizes two types of control: copy and create.<sup>§</sup>

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different pieces of authorization. A formal definition is given in Section 2.2.2.

subject's domain must carry the copy flag.

subject to the destination subject. In the definition of link predicates  $\{link_i\}$  for this model, a link exists from one subject to another provided the conditions are true in the given state.

with the link predicate  $link_i$  must also

link predicate from another as well as to control rights and filter functions. The symbol  $link_i$  has significance with respect to the control rights of the link. We now formally define

and operation called demand. Demand is not a control right to be formally redundant [37].

### 2.2.1 Link Predicates

In an SPM scheme a finite collection of link predicates is defined. Each predicate takes two subjects, say  $U$  and  $V$ , as arguments and evaluates to true or false. If true, it establishes a connection from  $U$  to  $V$  which can be used to copy tickets from  $U$  to  $V$ . The definition of each link predicate is in terms of the presence of some combination of control tickets for  $U$  and  $V$  in the domains of  $U$  and  $V$ . The idea is that link predicates can therefore be evaluated by examining the domains of the two subjects of concern and that too only with respect to presence of control tickets for these two subjects. That the definition should depend only on the presence and not the absence of tickets is a well-known principle for protection [31]. As a special case we also allow a link predicate which is always true to be defined. Formally we have the following definition.

**Definition 1** Let  $dom(U)$  be the set of tickets possessed by subject  $U$ . A local link predicate  $link_i(U,V)$  with  $U$  and  $V$  as formal parameters is defined as a conjunction or disjunction, but not negation, of the following terms for any  $z \in RC$ :  $U/z \in dom(U)$ ,  $U/z \in dom(V)$ ,  $V/z \in dom(U)$ ,  $V/z \in dom(V)$ , and true.

For a given state if  $link_i(A,B)$  is true we say there is a  $link_i$  from  $A$  to  $B$ . We emphasize the existence of a link is necessary but not sufficient for copying tickets from  $A$  to  $B$ . Examples of local link predicates from the literature are listed below.

1.  $link_{tg}(U,V) \equiv U/t \in dom(V) \vee V/g \in dom(U)$
2.  $link_t(U,V) \equiv U/t \in dom(V)$
3.  $link_g(U,V) \equiv V/g \in dom(U)$
4.  $link_{sr}(U,V) \equiv U/r \in dom(V) \wedge V/s \in dom(U)$
5.  $link_b(U,V) \equiv U/b \in dom(U)$
6.  $link_p(U,V) \equiv V/p \in dom(V)$
7.  $link_{bp}(U,V) \equiv U/b \in dom(U) \wedge V/p \in dom(V)$
8.  $link_u(U,V) \equiv \text{true}$

The first example is from the take-grant model [21] where the  $t$  and  $g$  control rights are respectively read as take and grant. The next two examples each retain just one of these privileges [23]. The fourth example is from the send-receive mechanism [28, 32] where the  $s$  and  $r$  control rights are respectively read as send and receive. The first four cases are defined in terms of control tickets for  $U$  in  $V$ 's domain or vice versa. The next three cases are quite different and are defined in terms of a control ticket for  $U$  in  $U$ 's domain or similarly for  $V$ . The last case is unique in that it requires no tickets for a link to exist. There are other interesting possibilities for defining link predicates. We anticipate that simple predicates of the kind defined above will suffice in practice, although the model does allow for arbitrarily complex ones.

Since SPM is a model and not an implementation, the precise mechanics by which a link is evaluated to be true or false are deliberately left unspecified. The conservative approach would be to evaluate a link on every occasion that a copy operation is attempted using that link. It is possible to have implementations where the link is evaluated once and "cached" to enable several copy operations. Similarly it is left unspecified whether subjects have to explicitly identify which link to use in a copy operation or whether the operating system will search for the existence of a suitable link. At this level of detail there are numerous alternatives consistent with the abstract SPM model.

### 2.2.2 Filter Functions

The final condition required for authorizing a copy operation is defined by the filter functions  $f_i: TS \times TS \rightarrow 2^{T \times R}$ , one for each predicate  $link_i$ . The interpretation is that  $Y/x:c$  can be copied from  $dom(U)$  to  $dom(V)$  if and only if all of the following are true for some  $link_i$ .

1.  $Y/x:c \in dom(U)$
2.  $link_i(U, V)$  evaluates to true
3.  $y/x:c \in f_i(u, v)$  where  $U, V$  and  $Y$  are of type  $u, v$  and  $y$  respectively

Some possible values of  $f_i(u, v)$  are  $T \times R$ ,  $TO \times RI$  and  $\phi$  respectively authorizing all tickets, inert tickets and no tickets to be copied from a subject of type  $u$  to a subject of type  $v$  over a link  $i$ .

The copy flag, link predicates and filter functions together authorize a copy operation in this manner. The first two conditions depend on the protection state whereas the third depends only on the scheme. Note that  $Y/x:c$  is required in  $dom(U)$  for copying either of  $Y/x:c$  or  $Y/x$ . The filter function determines whether or not the copied ticket can have the copy flag. Selectivity in copying is controlled by the filter function entirely in terms of types.

We emphasize that there is a different filter function  $f_i$  for each predicate  $link_i$ . Also, the value of a filter function  $f_i$  can specify a different set of ticket types for each pair of its argument subject types. Filter functions are a powerful tool for specifying policies. They impose non-discretionary controls which are inviolable and confine the discretionary behavior of individual subjects.

SPM imposes no assumptions regarding the role of  $U$  and  $V$  in a copy operation from  $U$  to  $V$ . For worst-case analysis it is equally acceptable that copying take place at the initiative of  $U$  or  $V$  alone or require both to cooperate. In this respect SPM is similar to HRU, which also does not specify which of the subjects involved in a command are regarded as initiators of the command.

### 2.3 The Create Operation

The create operation introduces new subjects and objects in the system. There are two issues here: what types of entities can be created and which tickets are introduced as the immediate result of a create operation.

#### 2.3.1 The Can-Create Function

Authorization for creation is specified by the function  $cc: TS \rightarrow 2^T$ . The interpretation is that a subject  $v$  can create entities of type  $v$  if and only if  $v \in cc(u)$ . Similarly,  $cc(security-domain)$  specifies the types of entities that can be created to create users.

#### 2.3.2 Create Rules

The tickets introduced by a create operation are defined by the function  $cr_p(u, v)$  for every pair  $(u, v)$  such that  $v \in cc(u)$ . The tickets introduced are for the parent and child. The motivation is that creation should not have an impact on the state. We emphasize that  $v \in cc(u)$ .

Let subject  $U$  of type  $u$  create entity  $V$  of type  $v$ . If  $V$  is an object the create-rule symbol signifying the created object.

$$cr_p(u, v) \subseteq \{$$

The interpretation is that the parent  $U$  creates the child  $V$ . For example,  $cr_p(user, file) = child/rwc$  signifying that the child can read and write tickets for it.

If  $V$  is a subject the situation is more complex. We specify tickets to be placed in  $V$ 's domain. The parent has two components as follows.

$$\begin{aligned} cr_p(u, v) &\subseteq \{child/ \\ cr_c(u, v) &\subseteq \{child/ \end{aligned}$$

These respectively specify tickets to be placed in the parent and child domains. Tickets for the parent and child are  $parent/x:c$  and  $child/y:c$  respectively. The interpretation is that the parent gets  $U/x:c$  provided  $parent/x:c \in cr_c(u, v)$  and  $V/y:c$  provided  $child/y:c \in cr_p(u, v)$ . The motivation for allowing a create-operation to create a subject in the parent's own domain is discussed at length in [1].

The following example from the taxonomy shows that a subject gets copiable take and grant tickets with an empty domain:  $cr_p(s, s) = child/take$ .

#### 2.4 Summary Of SPM

In summary, SPM requires the security policy to be defined by the following components:

1. A finite set of types  $T$  partitioned into subject types  $TS$  and object types  $TO$ .



plementation, the precise mechanics by which links are deliberately left unspecified. The need to create a link on every occasion that a copy operation is performed. It is possible to have implementations that "enable" to enable several copy operations. Subjects have to explicitly identify which link is used. The operating system will search for the link. In detail there are numerous alternatives

Authorizing a copy operation is defined by the filter function  $f_i$  for each predicate link  $l_i$ . The interpretation of  $f_i(U)$  to  $\text{dom}(V)$  if and only if all of the

of type  $u$ ,  $v$  and  $y$  respectively

TO  $\times$  RI and  $\phi$  respectively authorizing a copy operation to be copied from a subject of type  $u$  to a

Filter functions together authorize a copy operation. Conditions depend on the protection state of the protection scheme. Note that  $Y/x:c$  is required in the filter function. The filter function determines whether a copy operation is allowed. Selectivity in copying is controlled by the filter function.

Filter function  $f_i$  for each predicate link  $l_i$ . Filter functions specify a different set of ticket types for each predicate link. Filter functions are a powerful tool for implementing discretionary controls which are inviolable for individual subjects.

Understanding the role of U and V in a copy operation is equally acceptable that copying take operations require both to cooperate. In this respect, the filter function does not specify which of the subjects involved in the command.

Subjects and objects in the system. There are subjects that can be created and which tickets are required for a create operation.

### 2.3.1 The Can-Create Function

Authorization for creation is specified in a scheme by the can-create function  $cc : TS \rightarrow 2^T$ . The interpretation is that subjects of type  $u$  are authorized to create entities of type  $v$  if and only if  $v \in cc(u)$ . For example  $cc(\text{user}) = \{\text{file}\}$  authorizes users to create files. Similarly,  $cc(\text{security-officer}) = \{\text{user}\}$  authorizes security officers to create users.

### 2.3.2 Create Rules

The tickets introduced by a create operation are specified by a create-rule for every pair  $(u, v)$  such that  $v \in cc(u)$ . The create-rules are local in that the only tickets introduced are for the parent and child entities in the domains of these two entities. The motivation is that creation should immediately have only a local incremental impact on the state. We emphasize there is a different create-rule for each pair  $v \in cc(u)$ .

Let subject U of type  $u$  create entity V of type  $v$ , so U is the parent and V the child. If V is an object the create-rule is specified as follows, where *child* is a special symbol signifying the created object.

$$cr_p(u, v) \subseteq \{child/x:c \mid x:c \in RI\}$$

The interpretation is that the parent U gets  $V/x:c$  if and only if  $x:c \in cr_p(u, v)$ . For example,  $cr_p(\text{user}, \text{file}) = \text{child}/rwc$  specifies that the creator of a file gets copiable read and write tickets for it.

If V is a subject the situation is more complex since the create-rule must also specify tickets to be placed in V's domain. So if  $v$  is a subject type the create-rule has two components as follows.

$$\begin{aligned} cr_p(u, v) &\subseteq \{child/x:c, parent/x:c \mid x:c \in R\} \\ cr_c(u, v) &\subseteq \{child/x:c, parent/x:c \mid x:c \in R\} \end{aligned}$$

These respectively specify tickets to be placed in the parent and child domains. Tickets for the parent and child are identified by the special symbols *parent* and *child* respectively. The interpretation is the parent U gets  $U/x:c$  provided  $parent/x:c \in cr_p(u, v)$  and  $V/x:c$  provided  $child/x:c \in cr_p(u, v)$ . Similarly the child V gets  $U/x:c$  provided  $parent/x:c \in cr_c(u, v)$  and  $V/x:c$  provided  $child/x:c \in cr_c(u, v)$ . The motivation for allowing a create-rule to introduce tickets for the parent in the parent's own domain is discussed at length in [34].

The following example from the take-grant model [21] specifies that the parent subject gets copiable take and grant tickets for its child, while the child is created with an empty domain:  $cr_p(s, s) = \text{child}/rwc$  and  $cr_c(s, s) = \phi$ .

## 2.4 Summary Of SPM

In summary, SPM requires the security administrator to specify an authorization scheme by defining the following components.

1. A finite set of types T partitioned into disjoint sets of subject types TS and object types TO.

2. A finite set of rights  $R$  partitioned into disjoint sets of inert rights  $RI$  and control rights  $RC$ .
3. A finite collection of local link predicates  $\{\text{link}_i\}$ .
4. A filter function  $f_i: TS \times TS \rightarrow 2^{T \times R}$  for each predicate  $\text{link}_i$ .
5. A can-create function  $cc: TS \rightarrow 2^T$ .
6. A local create-rule for each  $(u, v)$  such that  $v \in cc(u)$ .

A system is specified by defining a scheme and the initial protection state, i.e., the initial set of entities and the initial distribution of tickets. Thereafter the state evolves by copy and create operations.

### 2.5 Revocation

SPM is monotonic in that it lacks facilities for revocation of tickets and deletion of entities. In any real system there must, of course, be mechanisms for revocation and deletion. Similarly, any implementation of SPM would also provide these mechanisms. Fortunately it turns out that under rather general assumptions revocation and deletion can be ignored for safety analysis in the worst case.

Revocation can be ignored in a worst-case scenario provided the effect of revocation can be undone. We call this the *restoration principle*, i.e., whatever can be revoked can be restored [34]. In SPM, if a ticket obtained by a copy operation is revoked it is easily restored by repeating the operation. However if a ticket introduced by a create operation is revoked, it may not be restorable by repeating the operation since each created entity is unique. Also tickets distributed in the initial state may not be restorable. If we assume tickets distributed in the initial state or introduced by create-rules are irrevocable, the restoration principle does not entail any loss of generality in context of SPM. The need for a restoration principle is also demonstrated by the lost object problem. With unrestricted revocation it is possible that all tickets for an object disappear. The object thereby becomes inaccessible.

The situation regarding deletion of entities is similar. Here the restoration principle requires that an entity which can be deleted should be replaceable by an equivalent entity. In general this rules out deletion of entities present in the initial state. Regarding deletion of entities created subsequently, it is always possible to re-create an entity of the same type as was deleted. In other words the individuality of created entities is not significant for analysis of the safety problem whereas the individuality of entities in the initial state may be significant.

To summarize, revocation and deletion policies which are consistent with the restoration principle can be ignored for analysis of the safety problem in a worst-case scenario.

### 2.6 An Example

We close this Section with a simple example of an SPM scheme based on the well-known concept of ownership. A user is regarded as the owner of all files created

by him and has complete discretion regarding the dynamics of the system. The scheme specifies this policy in SPM.

### Scheme 1 Basic owner-based policy.

1.  $TS = \{user\}$ ,  $TO = \{file\}$
2.  $RI = \{x:c\}$ ,  $RC = \phi$
3.  $\text{link}_u(X, Y) \equiv \text{true}$
4.  $f_u(user, user) = \{file/xc\}$
5.  $cc(user) = file$
6.  $cr_p(user, file) = \{file/xc\}$

The types *user* and *file* correspond to a single inert right  $x:c$  provides access regarding the dynamics of different in execute and append, remains the same universal link predicate is defined. Tickets can be copied across universal links. User for each created file.

Note that the specification  $f_u(user, user)$  behavior. In this case tickets given by carry the copy flag. Consequently the privileges for an owned file to other users.

## 3 Safety In SPM

In this Section we briefly review the propagation of access rights. For a state given, the safety problem asks whether the initial state, by the rules of the scheme. In analyzing this problem arises from the allow creation safety is easily determined simply keep executing copy operation further copy operations do not change the state the *no-creates maximal state*. To break the analysis problem into two parts

- I. From the initial state construct the state alone.
- II. Compute the no-creates maximal state.

This strategy works provided we have the mented state. We need to somehow account for the potential created.

two disjoint sets of inert rights  $RI$  and

sets  $\{link_i\}$ .

for each predicate  $link_i$ .

that  $v \in cc(u)$ .

and the initial protection state, i.e., the  
distribution of tickets. Thereafter the state

for revocation of tickets and deletion  
course, be mechanisms for revocation  
of SPM would also provide these mech-  
rather general assumptions revocation  
is in the worst case.

The scenario provided the effect of revo-  
cation principle, i.e., whatever can be  
ticket obtained by a copy operation is  
operation. However if a ticket intro-  
may not be restorable by repeating the  
Also tickets distributed in the initial  
tickets distributed in the initial state  
e, the restoration principle does not  
M. The need for a restoration principle  
blem. With unrestricted revocation it  
appear. The object thereby becomes

is similar. Here the restoration prin-  
deleted should be replaceable by an  
deletion of entities present in the initial  
subsequently, it is always possible to  
deleted. In other words the individuality  
ysis of the safety problem whereas the  
may be significant.

policies which are consistent with the  
ysis of the safety problem in a worst-

ple of an SPM scheme based on the  
regarded as the owner of all files created

by him and has complete discretion regarding access to these files. The following  
scheme specifies this policy in SPM.

**Scheme 1** *Basic owner-based policy.*

1.  $TS = \{user\}$ ,  $TO = \{file\}$
2.  $RI = \{x:c\}$ ,  $RC = \phi$
3.  $link_u(X,Y) \equiv \text{true}$
4.  $f_u(user, user) = \{file/xc\}$
5.  $cc(user) = file$
6.  $cr_p(user, file) = \{file/xc\}$

The types *user* and *file* correspond to users and files respectively. For simplicity,  
a single inert right  $x:c$  provides access to files. This suffices so long as the policy  
regarding the dynamics of different inert rights, such as the typical read, write,  
execute and append, remains the same. There are no control rights so only the  
universal link predicate is defined. Tickets for files, with or without the copy flag,  
can be copied across universal links. Users can create files and get a copiable ticket  
for each created file.

Note that the specification  $f_u(user, user) = \{file/x\}$  would give us a very different  
behavior. In this case tickets given by the owner of a file to other users cannot  
carry the copy flag. Consequently the owner is the only one who can ever grant  
privileges for an owned file to other users.

### 3 Safety In SPM

In this Section we briefly review the safety analysis of SPM with respect to  
propagation of access rights. For a system, whose initial state and scheme are  
given, the safety problem asks whether or not there is a state reachable from the  
initial state, by the rules of the scheme, with  $V/x:c$  in  $\text{dom}(U)$ . The complication  
in analyzing this problem arises from the create operation. If the scheme does not  
allow creation safety is easily determined by a polynomial time algorithm [34]. We  
simply keep executing copy operations until the state stabilizes in the sense that  
further copy operations do not change any subject's domain. We call this stable  
state the *no-creates maximal state*. Our approach to dealing with creation is to  
break the analysis problem into two phases, as follows.

- I. From the initial state construct an *augmented state* by create operations  
alone.
- II. Compute the no-creates maximal state from the augmented state of phase I.

This strategy works provided we have a method for constructing a suitable aug-  
mented state. We need to somehow prove that subjects and objects in the aug-  
mented state account for the potentially unbounded set of entities which can be  
created.

There is a very natural restriction under which the above strategy can be proved correct. Define the *cc*-digraph to be the directed graph with vertices  $T$  and an edge from  $u$  to  $v$  if and only if  $v \in cc(u)$ . We say *cc* is *acyclic* if this graph is acyclic. For acyclic *cc* we compute the augmented state in phase I as follows.

```

procedure augment
  mark all subjects in the initial state to be open;
  while there exists an open subject U do
    forall  $v \in cc(type(U))$  do
      let U create an entity of type  $v$ ;
      mark this created entity to be open;
    end
    mark U to be closed;
  end

```

It is obvious that this procedure terminates if and only if *cc* is acyclic. In [34] it is shown that the no-creates maximal state obtained from this augmented state correctly answers the safety question. The reason for this is quite simple. If a subject creates two entities of the same type, there is no difference between them as far as the scheme is concerned. So for purpose of safety analysis it suffices to create just one of them.

Of course if *cc* has cycles the above procedure will not terminate. Indeed it has been shown that with arbitrary cycles in *cc* safety is undecidable [38]. So there is no algorithm for computing a suitable augmented state in general. For the most part it appears that cycles in *cc* do not arise in practice. In our experience cycles in can-create occur only in the very special, but also very important, case where a subject is authorized to create new subjects of its own type. Such cycles are called loops and show up in the form  $u \in cc(u)$ . In other words loops are cycles of length one in the *cc*-digraph. The augmenting construction for *cc* with loops is as follows.

```

procedure augment with loops
  eliminate loops from cc;
  perform the augment procedure;
  restore the loops in cc;
  forall subjects U such that  $type(U) \in cc(type(U))$  do
    let U create a subject of type  $type(U)$ ;
  end

```

In [34] it is proved that the no-creates maximal state obtained from this augmented state correctly answers the safety question provided the create-rules for loops in *cc* satisfy the following restriction.

**Definition 2** A create-rule for a loop in *cc* is said to be attenuating if

1.  $cr_c(u, u) \subseteq cr_p(u, u)$
2.  $child/x : c \in cr_p(u, u) \Rightarrow parent/x : c \in cr_p(u, u)$

The motivation and justification for th

For our purpose in this paper, it is i  
sumed by SPM are indeed subsumed  
defined as follows:

**Definition 3** An SPM scheme is acycl  
has loops with attenuating create-rules.

To summarize, safety is decidable for a  
decision procedures given above are ef  
This completes our review of SPM.

#### 4 Equivalence Of Systems

Our main objective in this paper is  
known protection models as special cas  
define what we mean by subsumption.

**Definition 4** We say that SPM subsum  
tem  $S$  which can be specified in  $M$  we  
 $S'$ .

To complete this definition we need to  
simplest definition of equivalence is per

**Definition 5** Two systems  $S$  and  $S'$   
construct a mapping  $\sigma$  such that subject  
only if subject  $\sigma(s)$  can have access  $\sigma(r)$

By this definition systems are equivalent  
Note that there may be substantial diffe  
 $S'$ . For example:

1. Subject  $\sigma(s)$  may have to go thro  
 $\sigma(r)$  access to object  $\sigma(o)$ , as comp  
access  $r$  to object  $o$ .
2. There may be additional subjects,  
counterpart in  $S$ , but are present o  
of  $S$  by  $S'$ .
3. The mapping  $\sigma$  may be extremely

Nevertheless, from a perspective of w  
are equivalent. In other words if both  
cooperating Trojan Horses who are de  
as possible, the net accesses in both sy  
analysis of  $S$  reduces to safety analysis

for which the above strategy can be proved. The graph with vertices  $T$  and an edge  $cc$  is *acyclic* if this graph is acyclic. For the proof in phase I as follows.

to be open;  
do

of type  $v$ ;  
to be open;

if and only if  $cc$  is acyclic. In [34] it is shown that the state obtained from this augmented state is the reason for this is quite simple. If a system is of type  $v$ , there is no difference between them. For the purpose of safety analysis it suffices to

the procedure will not terminate. Indeed it has been shown that  $cc$  safety is undecidable [38]. So there is no algorithm to augment the state in general. For the most common case in practice. In our experience cycles are common, but also very important, case where a system has cycles of its own type. Such cycles are called self-loops. In other words loops are cycles of length 1. The construction for  $cc$  with loops is as follows.

$\in cc(\text{type}(U))$  do  
of type  $U$ ;

minimal state obtained from this augmented state is provided the create-rules for loops in  $cc$

$cc$  is said to be attenuating if

$cr_p(u, u)$

The motivation and justification for this definition are discussed at length in [34].

For our purpose in this paper, it is important to demonstrate that models subsumed by SPM are indeed subsumed by acyclic attenuating schemes, which are defined as follows:

**Definition 3** An SPM scheme is acyclic attenuating if its  $cc$ -digraph is acyclic or has loops with attenuating create-rules.

To summarize, safety is decidable for acyclic attenuating schemes. Moreover, the decision procedures given above are efficient unless the  $cc$ -digraph is very dense. This completes our review of SPM.

#### 4 Equivalence Of Systems

Our main objective in this paper is to show that SPM subsumes three well-known protection models as special cases. In order to do this we must of course define what we mean by subsumption. We do so as follows:

**Definition 4** We say that SPM subsumes a model  $M$  provided that for every system  $S$  which can be specified in  $M$  we can construct an equivalent SPM system  $S'$ .

To complete this definition we need to define the meaning of "equivalent." The simplest definition of equivalence is perhaps the following one.

**Definition 5** Two systems  $S$  and  $S'$  are said to be equivalent provided we can construct a mapping  $\sigma$  such that subject  $s$  can have access  $r$  to object  $o$  in  $S$  if and only if subject  $\sigma(s)$  can have access  $\sigma(r)$  to object  $\sigma(o)$  in  $S'$ .

By this definition systems are equivalent if they have equivalent worst case behavior. Note that there may be substantial differences between the details of system  $S$  and  $S'$ . For example:

1. Subject  $\sigma(s)$  may have to go through far more convoluted actions to acquire  $\sigma(r)$  access to object  $\sigma(o)$ , as compared to the actions of subject  $s$  in acquiring access  $r$  to object  $o$ .
2. There may be additional subjects, objects and rights in  $S'$  that have no direct counterpart in  $S$ , but are present due to bookkeeping details in the simulation of  $S$  by  $S'$ .
3. The mapping  $\sigma$  may be extremely complex (although it must be computable).

Nevertheless, from a perspective of worst-case safety analysis, the two systems are equivalent. In other words if both systems are assumed to be infested with cooperating Trojan Horses who are determined to propagate access rights as far as possible, the net accesses in both systems will be identical. Therefore, safety analysis of  $S$  reduces to safety analysis of  $S'$  and vice versa.

The constructions of this paper show that the Bell-LaPadula multilevel security model [3], take-grant models [16, 21, 41] and grammatical protection systems [10, 22] are all equivalent to acyclic attenuating schemes in SPM. Therefore, the safety analysis results of SPM also apply to these models. We reiterate that these three models, and SPM itself, are all subsumed by monotonic HRU. However, all known constructions of these models within HRU require multi-conditional commands (i.e., commands whose conditions have two or more terms), whereas safety is undecidable in HRU even for bi-conditional commands (i.e., commands whose conditions have exactly two terms).

The actual constructions given in this paper establish equivalence in a stronger sense than definition 5. It is beyond the scope of this paper to give a formal definition of "stronger" in this context. The intuition is that "stronger" means "behavioral equivalence." That is, every state transition in  $S$ , say from state  $\alpha$  to state  $\beta$ , can be mimicked by one or more state transitions in  $S'$  which applied to state  $\sigma(\alpha)$  result in state  $\sigma(\beta)$ . In other words, it is not only the states of  $S$  which are being simulated in  $S'$  but also the individual transitions. It will be evident that this requirement is easily satisfied by the constructions of this paper. Most equivalence results in computer science are actually behavioral equivalence results. For example, the familiar equivalence between classes of automata and formal grammars is of the behavioral variety.

Consider an example to make this intuition clearer. The take-grant model has decidable safety, therefore it is trivial to give an equivalent HRU system  $S'$  for a given take-grant system  $S$ . We simply run the safety algorithm of take-grant as part of the  $\sigma$  mapping, and construct the worst-case state as the target HRU system. These two systems are therefore worst-case equivalent with respect to safety. However, they are not behaviorally equivalent. In behavioral equivalence we are looking for simulation of actual behavior, so that what transpires in one system is accurately mimicked in the other. In other words behavioral equivalence requires equivalence of actual behavior, whereas definition 5 only requires equivalence of worst-case behavior.

## 5 Multilevel Security Models

The Bell-LaPadula (BLP) model [3] is a well-known access control model for multilevel security policies, most often applied in the military. In this Section we show how BLP is subsumed by SPM. This construction demonstrates that the traditional black-and-white distinction between mandatory and discretionary controls in the Bell-LaPadula model, has an alternate expression in SPM in terms of constraints on the propagation of access rights. The SPM viewpoint has the advantage of providing explicit machinery for formulating policies "in between" these two extremes.<sup>†</sup>

There has been recent controversy about exactly what the rules of BLP are [5, 24, 25, 27]. Moreover, since its original publication the model has been modified and reformulated in several ways in its application to specific design and implementation

<sup>†</sup>BLP can be extended to accommodate these "in between" policies, such as done in [26]. The point is that BLP needs to be extended for this purpose, whereas SPM already has the necessary mechanisms.

projects [18, 19]. Nevertheless most versions of SPM have in common that there is a clearly identifiable common

The key component in all versions of SPM is the security lattice derived from the military classification system. Each object in the access matrix is assigned a level from this lattice. The access matrix is an  $m \times n$  matrix, and the model specifies rules for how access rights are propagated. Rules are open ended, in the sense that they might be (other than that it requires a security level) and transforms the access matrix to a new one. This process involves constraints on the relative security levels of the objects to that operation. The controversy about the nature of the rules, since rules that allow a subject to change the security level of an object are troublesome in this respect. Versions of SPM that have been used are constant and cannot be changed after they are defined. Most practically used versions of the model have a tranquility requirement is slightly relaxed. This relaxation is effected by some designated security officials. Such relaxations of tranquility within SPM are common.

In this Section we consider two versions of SPM with different tranquility requirements. Our first version is due to Bellarelli [30] who showed that the BLP model is subsumed. We show that with tranquility this model can be expressed in SPM.

**Definition 6** *The BLP model with tranquility*

1.  $\Sigma = \{S_1, \dots, S_m\}$ , the set of subjects
2.  $\Omega = \{O_1, \dots, O_n\}$ , the set of objects
3.  $\Lambda = \{\lambda_1, \dots, \lambda_o\}$ , the lattice of security levels
4.  $\lambda: \Sigma \cup \Omega \rightarrow \Lambda$ , the current security levels
5.  $\lambda_{\max}: \Sigma \rightarrow \Lambda$ , the maximum security level
6.  $\lambda_{\max}(S_i), \lambda(O_j)$  are constants (tranquility levels)
7.  $R = \{r, w, o\}$ , the set of access rights
8. An  $m \times n$  discretionary access matrix and a set of discretionary access rights of  $S_i$  to  $O_j$ .
9. Subject  $S_i$  can create object  $O_j$  with security level  $\lambda(O_j)$ , that is the creation  $o \in M[i, j]$ .
10. The owner of an object can give access rights to another subject. That is, if  $o \in M[i, j]$  then  $o \in M[k, j]$ .

that the Bell-LaPadula multilevel security [1, 41] and grammatical protection system attenuating schemes in SPM. Therefore, they apply to these models. We reiterate that they are subsumed by monotonic HRU. However, they are within HRU require multi-conditional commands (i.e., commands whose conditions have two or more terms), whereas safety policies are conditional commands (i.e., commands whose

paper establish equivalence in a stronger sense than the scope of this paper to give a formal definition. The intuition is that "stronger" means that a state transition in  $S$ , say from state  $\alpha$  to state  $\beta$ , is subsumed by more state transitions in  $S'$  which applied to  $\alpha$ . In other words, it is not only the states of  $S$  but also the individual transitions. It will be clarified by the constructions of this paper. Behavioral equivalence are actually behavioral equivalence between classes of automata and systems.

The definition is clearer. The take-grant model has been used to give an equivalent HRU system  $S'$  for any system  $S$ . We may run the safety algorithm of take-grant on  $S$  to reach the worst-case state as the target HRU system. The take-grant model is worst-case equivalent with respect to behavioral equivalence. In behavioral equivalence we require that what transpires in one system is also transpired in the other. In other words behavioral equivalence requires that the two systems are behaviorally equivalent. In definition 5 only requires equivalence of

BLP is a well-known access control model for which has been applied in the military. In this Section we will show that BLP can be expressed as an SPM. This construction demonstrates that there is an equivalence between mandatory and discretionary access control. An alternate expression in SPM in terms of access rights. The SPM viewpoint has the advantage of providing a framework for formulating policies "in between"

that exactly what the rules of BLP are [5, 24]. In this Section we will show that the model has been modified and adapted to specific design and implementation

of "in between" policies, such as done in [26]. The purpose, whereas SPM already has the necessary

projects [18, 19]. Nevertheless most versions of the model are closely related and there is a clearly identifiable common core.

The key component in all versions of BLP is a lattice of *security levels*, usually derived from the military classification system [12, 18]. Each subject and object is assigned a level from this lattice. Access rights are represented in an access matrix, and the model specifies rules by which this matrix can be modified. The rules are open ended, in the sense there is no formal constraint on what a rule might be (other than that it requires authorization by the current access matrix and transforms the access matrix to a new state). In practice the rules typically involve constraints on the relative security levels of subjects and objects pertaining to that operation. The controversy about the model stems from the open-ended nature of the rules, since rules that are intuitively insecure can be defined [24]. Rules which change the security levels of subjects and objects are particularly troublesome in this respect. Versions of the model in which these security levels are constant and cannot be changed are said to satisfy the *tranquility* requirement. Most practically used versions of the model do require tranquility. Sometimes the tranquility requirement is slightly relaxed to allow changes in security levels to be effected by some designated security officer. We shall examine how to accommodate such relaxations of tranquility within SPM at the end of this Section.

In this Section we consider two versions of the BLP model, both with strong tranquility requirements. Our first version, defined below, is adapted from Pittelli [30] who showed that the BLP model he considered is an instance of HRU. We show that with tranquility this model can be expressed as an SPM scheme.

**Definition 6** *The BLP model with tranquility defines a system as follows.*

1.  $\Sigma = \{S_1, \dots, S_m\}$ , the set of subjects.
2.  $\Omega = \{O_1, \dots, O_n\}$ , the set of objects where  $\Sigma \cap \Omega = \phi$ .
3.  $\Lambda = \{\lambda_1, \dots, \lambda_o\}$ , the lattice of security levels with dominance relation  $\supseteq$ .
4.  $\lambda: \Sigma \cup \Omega \rightarrow \Lambda$ , the current security level of subjects and objects.
5.  $\lambda_{\max}: \Sigma \rightarrow \Lambda$ , the maximum security level of subjects (i.e.,  $\lambda_{\max}(S_i) \supseteq \lambda(S_i)$ ).
6.  $\lambda_{\max}(S_i), \lambda(O_j)$  are constants (tranquility).
7.  $R = \{r, w, o\}$ , the set of access rights (read, write, own).
8. An  $m \times n$  discretionary access matrix  $M$ , with  $M[i,j] \subseteq R$  specifying the discretionary access rights of  $S_i$  to  $O_j$ .
9. Subject  $S_i$  can create object  $O_j$  with arbitrary  $\lambda(O_j)$ . Immediately after creation  $o \in M[i,j]$ , that is the creator is the owner of the object.
10. The owner of an object can give read and write access to that object to another subject. That is, if  $o \in M[i,j]$  then  $S_i$  can enter  $r$  or  $w$  in  $M[k,j]$  for any  $k$ .

11. An  $m \times n$  current access matrix  $B$ , with  $B[i,j] \subseteq \{r,w\}$ , specifying the current access rights of  $S_i$  to  $O_j$  determined for  $r$  and  $w$  as follows.<sup>||</sup>

$$\begin{array}{ll} r \in B[i,j] & \Leftrightarrow r \in M[i,j] \wedge \lambda(S_i) \supseteq \lambda(O_j) \quad \text{Simple security} \\ w \in B[i,j] & \Leftrightarrow w \in M[i,j] \wedge \lambda(S_i) \sqsubseteq \lambda(O_j) \quad \text{Star-property} \end{array}$$

There is a fixed set of subjects. The current security level of a subject is given by  $\lambda$  and can change so long as it is dominated by the subject's maximum security level given by  $\lambda_{\max}$ . Subjects are allowed to create objects, and on doing so the creator becomes the owner of the created object. Each object has a security level assigned at the time of creation and given by  $\lambda$ . Tranquility implies that this level cannot change. Versions of BLP without tranquility usually have a security officer subject who can change  $\lambda$  for objects and possibly  $\lambda_{\max}$  for subjects. The potential dangers of unrestrained non-tranquility are demonstrated in [24]. The owner of an object has discretion regarding who may access that object. However, access can be exercised only if it is consistent with simple security and the star-property. The star-property is also called the confinement property. Sometimes append and execute rights are also defined. We have dropped these for simplicity, since these could be handled in much the same way as read and write in our construction.

Before proceeding further it is worth clarifying a point of terminology. What we are calling "security levels" or simply "levels," that is the elements of the security lattice, are often called "classifications" or "clearances" in the literature. The term "classification" is typically used for objects while subjects have "clearances." Moreover, military classifications are derived by combining a linearly ordered level and a set of compartments or caveats. The security lattice is derived by combining the linear ordering on levels with the subset relation on compartments. In the formalism it is irrelevant how the elements of the lattice are derived, so we can simply begin with a given lattice whose elements we call levels. Actually, the levels need not even constitute a lattice. For multilevel access control models it suffices that the levels be partially ordered.

One difficulty in constructing an SPM scheme equivalent to BLP is that the current security level of a BLP subject can change, resulting in changes in the current access matrix  $B$ . Lowering  $\lambda(S_i)$  shrinks the set of objects that  $S_i$  can read while expanding the set of objects which  $S_i$  can write. Similarly raising  $\lambda(S_i)$  expands the set of objects that  $S_i$  can read while shrinking the set of objects which  $S_i$  can write. This non-monotonic behavior implies that a BLP subject cannot be modeled as a single SPM subject. We circumvent this problem by mapping a BLP subject with varying  $\lambda$  to a set of SPM subjects, each with a fixed security level. Specifically the BLP subject  $S_i$  is mapped to the set  $\{S_i\lambda_1 \mid \lambda_{\max}(S_i) \supseteq \lambda_1\}$  of SPM subjects. Each SPM subject  $S_i\lambda_1$  has the fixed security level  $\lambda_1$  and is of the SPM type  $\sigma\lambda_1$ . The idea is that  $S_i\lambda_1$  simulates the BLP subject  $S_i$  when  $\lambda(S_i) = \lambda_1$ . SPM subjects derived from the same BLP subject in this manner are said to be *cohorts*.

<sup>||</sup>In the original BLP formulation  $B[i,j]$  is a subset of  $M[i,j]$  as defined here. This is because rights are entered in  $B$  only as per the actual accesses attempted by subjects. In other words the "if and only if" ( $\Leftrightarrow$ ) in the two conditions enumerated here is actually an "only if" ( $\Rightarrow$ ) in the original BLP model [3]. The "if and only if" formulation we have chosen is slightly simpler to deal with, although it is possible to simulate the original BLP "only if" formulation if so desired.

The connection between cohorts is made for every pair of cohorts. This allows ownership to be shared among the SPM cohorts.

In our construction each BLP object is a BLP object with current security level  $\lambda$  and maximum security level  $\lambda_{\max}$ . These SPM subjects are passive. The reason they are subjects is they act for themselves. These "self tickets" are used to allow us, for instance, to conveniently create an object can obtain read and write tickets for itself and the star-property are not violated.

The second difficulty in our construction is that  $S_i$  to create a BLP object  $O_j$  with arbitrary security level  $\lambda(O_j)$  and maximum security level  $\lambda_{\max}(S_i) \not\supseteq \lambda(O_j)$ . By simple security  $\lambda(O_j) \supseteq \lambda(S_i)$  for  $O_j$  to other subjects. In our simulation  $S_i\lambda_1$ , should be able to give read access to  $O_j$ . But this requires that  $S_i\lambda_1$  possess the right to  $O_j$ , contrary to simple security. The limitation of the SPM copy operation is that the right before that right can be given to  $S_i\lambda_1$  in the following situations where  $S_i$  is the

1. Let  $\lambda_{\max}(S_i) \supseteq \lambda(O_j)$  where  $\lambda(O_j) \supseteq \lambda(S_i)$ . By simple security, if  $\lambda(O_j) \supseteq \lambda(S_i)$  then  $S_i$  can give read access to  $O_j$ . However as the owner of  $O_j$   $S_i$  can give access for  $O_j$  to other subjects.
2. Let  $\lambda_{\max}(S_i) \supseteq \lambda(O_j)$ . By the star-property,  $S_i$  should not be able to give write access to  $O_j$ . However  $S_i\lambda_1$  should have the ability to give write access to  $O_j$ .

There is an elegant, and quite general, way to overcome this apparent limitation of its copy operation. The key idea is to introduce new rights for SPM subjects. The ability to give read and write access to other subjects. These symbols are converted to  $r$  and  $w$  respectively in the star-property.

These considerations lead us to define

## Scheme 2 BLP with tranquility.

1.  $TS = \{\sigma\lambda_i, o\lambda_i \mid \lambda_i \in \Lambda\}$ ,  $TO = \{o\lambda_i \mid \lambda_i \in \Lambda\}$ ,  $RI = \{r:c, w:c\}$ ,  $RC = \{o:c, l:c\}$ . SPM subjects of type  $\sigma\lambda_i$  simulate BLP subjects of type  $\lambda_i$ . SPM subjects of type  $o\lambda_i$  simulate BLP objects of type  $\lambda_i$ .
2.  $RI = \{r:c, w:c\}$ ,  $RC = \{o:c, l:c\}$ .



with  $B[i,j] \subseteq \{r,w\}$ , specifying the current security level for  $r$  and  $w$  as follows.<sup>11</sup>

$(S_i) \sqsupseteq \lambda(O_j)$  Simple security  
 $(S_i) \sqsubseteq \lambda(O_j)$  Star-property

The current security level of a subject is given by the subject's maximum security level. To create objects, and on doing so the subject. Each object has a security level  $\lambda$ . Tranquility implies that this level usually have a security officer possibly  $\lambda_{\max}$  for subjects. The potential demonstrated in [24]. The owner of an object has access to that object. However, access can be granted under simple security and the star-property. The star-property. Sometimes append and drop these for simplicity, since these are not needed for read and write in our construction.

Defining a point of terminology. What we mean by "clearances" that is the elements of the security lattice or "clearances" in the literature. The objects while subjects have "clearances." This is achieved by combining a linearly ordered level security lattice is derived by combining a subset relation on compartments. In the levels of the lattice are derived, so we can call them compartments we call levels. Actually, the levels of multilevel access control models it suffices

A scheme equivalent to BLP is that the security level can change, resulting in changes in the security level which shrinks the set of objects that  $S_i$  can read and which  $S_i$  can write. Similarly raising  $\lambda(S_i)$  while shrinking the set of objects which  $S_i$  can write implies that a BLP subject cannot be allowed to write. To prevent this problem by mapping a BLP subject to a set of SPM subjects, each with a fixed security level. The set of SPM subjects is  $\{S_i \lambda_i \mid \lambda_{\max}(S_i) \sqsupseteq \lambda_i\}$  of SPM subjects with fixed security level  $\lambda_i$  and is of the SPM subject  $S_i$  when  $\lambda(S_i) = \lambda_i$ . SPM subjects created in this manner are said to be *cohorts*.

A subset of  $B[i,j]$  as defined here. This is because the security levels attempted by subjects. In other words the security level granted here is actually an "only if" ( $\Rightarrow$ ) in the simulation we have chosen is slightly simpler to the original BLP "only if" formulation if so desired.

The connection between cohorts is maintained by setting up cohort links between every pair of cohorts. This allows ownership of an object created by a BLP subject to be shared among the SPM cohorts for that BLP subject.

In our construction each BLP object is mapped to an SPM subject. Specifically, a BLP object with current security level  $\lambda_1$  is mapped to an SPM subject of type  $o\lambda_1$ . These SPM subjects are passive entities which cannot initiate any operations. The reason they are subjects is they possess tickets with the  $rc$  and  $wc$  rights for themselves. These "self tickets" are useful at various places in the simulation. They allow us, for instance, to conveniently specify that the SPM cohorts that own an object can obtain read and write tickets for that object, provided simple security and the star-property are not violated.

The second difficulty in our construction arises from the ability of a BLP subject  $S_i$  to create a BLP object  $O_j$  with arbitrary  $\lambda(O_j)$ . Now consider what happens if  $\lambda_{\max}(S_i) \not\sqsupseteq \lambda(O_j)$ . By simple security the creator  $S_i$  cannot read the created object  $O_j$ . However by virtue of being the owner,  $S_i$  has the ability to give read access for  $O_j$  to other subjects. In our simulation at least one of the SPM cohorts of  $S_i$ , say  $S_i \lambda_1$ , should be able to give read access to  $O_j$  to cohorts of other BLP subjects. But this requires that  $S_i \lambda_1$  possess the  $O_j/rc$  ticket and thereby have read access to  $O_j$ , contrary to simple security. This situation appears to indicate an inherent limitation of the SPM copy operation, i.e., SPM requires a subject to possess a right before that right can be given to another subject. Similar problems arise in the following situations where  $S_i$  is the creator of  $O_j$ .

1. Let  $\lambda_{\max}(S_i) \sqsupseteq \lambda(O_j)$  where  $\lambda(O_j)$  is not the bottom element of the lattice. By simple security, if  $\lambda(O_j) \sqsupseteq \lambda(S_i) = \lambda_1$ , the SPM cohort  $S_i \lambda_1$  should not be able to read  $O_j$ . However as the owner  $S_i \lambda_1$  should have the ability to give read access for  $O_j$  to other subjects.
2. Let  $\lambda_{\max}(S_i) \sqsupseteq \lambda(O_j)$ . By the star-property, if  $\lambda(S_i) = \lambda_1 \sqsupseteq \lambda(O_j)$ , the SPM cohort  $S_i \lambda_1$  should not be able to write  $O_j$ . However as the owner  $S_i \lambda_1$  should have the ability to give write access for  $O_j$  to other subjects.

There is an elegant, and quite general, technique by which SPM gets around this apparent limitation of its copy operation, by means of links and filter functions. The key idea is to introduce new right symbols  $\hat{r}:c$  and  $\hat{w}:c$ , which control the ability to give read and write access respectively to other subjects. These right symbols are converted to  $r$  and  $w$  respectively if so allowed by simple security and the star-property.

These considerations lead us to define the following scheme.

### Scheme 2 BLP with tranquility.

1.  $TS = \{\sigma\lambda_i, o\lambda_i \mid \lambda_i \in \Lambda\}$ ,  $TO = \phi$   
 SPM subjects of type  $\sigma\lambda_i$  simulate BLP subjects with current level  $\lambda_i$ . SPM subjects of type  $o\lambda_i$  simulate BLP objects with current level  $\lambda_i$ .
2.  $RI = \{r:c, w:c\}$ ,  $RC = \{o:c, k:c, \hat{r}:c, \hat{w}:c\}$

$r$ ,  $w$ , and  $o$  are the original BLP rights;  $k$ ,  $\hat{r}$  and  $\hat{w}$  are artifacts of the simulation

3.  $\text{link}_u(U, V) \equiv \text{true}$   
 $\text{link}_o(U, V) \equiv U/o \in \text{dom}(V)$   
 $\text{link}_k(U, V) \equiv U/k \in \text{dom}(V) \wedge V/k \in \text{dom}(U)$   
 $\text{link}_{\hat{r}}(U, V) \equiv U/\hat{r} \in \text{dom}(V)$   
 $\text{link}_{\hat{w}}(U, V) \equiv U/\hat{w} \in \text{dom}(V)$

The subscripts on these links have the following mnemonic significance:  $u$  for universal,  $o$  for owner,  $k$  for cohort,  $\hat{r}$  for discretionary read access, and  $\hat{w}$  for discretionary write access.

4. Undefined values of the filter functions are assumed by default to be  $\phi$ .

$$f_u(\sigma\lambda_i, \sigma\lambda_j) = \{o\lambda_i/\hat{r}\hat{w} \mid \lambda_i \in \Lambda\}$$

$$f_k(\sigma\lambda_i, \sigma\lambda_j) = \{o\lambda_i/oc \mid \lambda_i \in \Lambda\}$$

$$f_o(o\lambda_i, \sigma\lambda_j) = o\lambda_i/\hat{r}\hat{w}c$$

$$f_{\hat{r}}(o\lambda_i, \sigma\lambda_j) = \text{if } \lambda_i \sqsubseteq \lambda_j \text{ then } o\lambda_i/r \text{ else } \phi$$

$$f_{\hat{w}}(o\lambda_i, \sigma\lambda_j) = \text{if } \lambda_i \supseteq \lambda_j \text{ then } o\lambda_i/w \text{ else } \phi$$

5.  $cc(\sigma\lambda_i) = \{o\lambda_j \mid \lambda_j \in \Lambda\}$

6.  $cr_p(\sigma\lambda_i, o\lambda_j) = \text{child}/oc$ ,  
 $cr_c(\sigma\lambda_i, o\lambda_j) = \text{child}/r\hat{w}\hat{w}c$

The simple security and star properties of BLP are respectively enforced by  $f_{\hat{r}}$  and  $f_{\hat{w}}$ . Discretionary control over access to a created entity is enforced by  $f_k$ ,  $f_o$ ,  $f_u$  and the create-rules. Note that  $cc$  is acyclic and sparse so safety for this scheme is decidable in polynomial time by the technique of Section 3. This is in contrast to Pittelli's instantiation of BLP in HRU [30] where the resulting HRU system does not fall within the decidable cases of HRU (because Pittelli's construction uses multi-conditional commands).

To complete the construction we define the initial state to be as follows, where SUB is the initial set of subjects in the SPM system.

1.  $\text{SUB} = \Omega \cup \{S_i \lambda_i \in \Sigma \times \Lambda \mid \lambda_{\max}(S_i) \supseteq \lambda_i\}$
2.  $\text{type}(O_j) = o\lambda_j$ , where  $\lambda(O_j) = \lambda_j$   
 $\text{type}(S_i \lambda_i) = \sigma\lambda_i$ , where  $\lambda_{\max}(S_i) \supseteq \lambda_i$
3.  $\text{dom}(O_i) = O_i/r\hat{w}\hat{w}c$

$$\text{dom}(S_i \lambda_j) = \{S_i \lambda_i/k \mid S_i \lambda_i \in \text{SUB}\} \cup \{O_i/o\hat{r}\hat{w}c \mid o \in M[i,1]\} \cup$$

$$\{O_i/\hat{r} \mid r \in M[i,1]\} \cup \{O_i/\hat{w} \mid w \in M[i,1]\} \cup$$

$$\{O_i/r \mid r \in B[i,1]\} \cup \{O_i/w \mid w \in B[i,1]\}$$

Each subject gets the  $k$  right for all its cohorts. The entries of the BLP M matrix are represented by the  $o$ ,  $\hat{r}$  and  $\hat{w}$  rights. The entries of the BLP B matrix are represented by the  $r$  and  $w$  rights.

Operation	BLP System [Let $\lambda(S_i) = \lambda_p$ , $\lambda(O_j) =$
1	$S_i$ creates $O_j$
Net Effect	$o \in M[i,j]$
2	$S_i$ enters $r$ in $M[k,j]$
Net Effect	$r \in M[k,j]$ $r \in B[k,j]$ if $\lambda(S_k) \supseteq \lambda_q$
3	$S_i$ enters $w$ in $M[k,j]$
Net Effect	$w \in M[k,j]$ $w \in B[k,j]$ if $\lambda(S_k) \sqsubseteq \lambda_q$
4	$S_i$ changes $\lambda(S_i)$
Net Effect	Row $i$ of $B$ is changed to serve simple-security and star-property.

Table 1: Simulation of

rights;  $k$ ,  $\hat{r}$  and  $\hat{w}$  are artifacts of the

$\text{dom}(U)$

the following mnemonic significance:  $u$  for discretionary read access, and  $\hat{w}$  for

operations are assumed by default to be  $\phi$ .

use  $\phi$   
else  $\phi$

BLP are respectively enforced by  $f_{\hat{r}}$  and  $f_{\hat{w}}$ . The created entity is enforced by  $f_k$ ,  $f_o$ ,  $f_u$  and sparse so safety for this scheme is unique of Section 3. This is in contrast to [1] where the resulting HRU system does not enforce HRU (because Pittelli's construction uses

the initial state to be as follows, where  $\lambda_1$  is the SPM system.

$\lambda_1$

$$\{O_i/o\hat{r}\hat{w}c \mid o \in M[i,l]\} \cup \{O_i/\hat{w} \mid w \in M[i,l]\} \cup \{O_i/w \mid w \in B[i,l]\}$$

cohorts. The entries of the BLP M matrix are  $M[k,j]$ . The entries of the BLP B matrix are  $B[k,j]$ .

Operation	BLP System [Let $\lambda(S_i)=\lambda_p, \lambda(O_j)=\lambda_q$ ]	SPM Simulation
1	$S_i$ creates $O_j$	$S_i \lambda_p$ creates $O_j$ of type $o\lambda_q$ and copies $O_j/oc$ to its cohorts $S_i \lambda_1$ over $\text{link}_k$ . Each cohort copies $O_j/\hat{r}\hat{w}c$ from $O_j$ over $\text{link}_o$ .
Net Effect	$o \in M[i,j]$	$O_j/o\hat{r}\hat{w}c \in \text{dom}(S_i \lambda_1)$ for all $\lambda_1$
2	$S_i$ enters $r$ in $M[k,j]$	$S_i \lambda_p$ copies $O_j/\hat{r}$ over $\text{link}_u$ to all cohorts of $S_k$ . Each cohort $S_k \lambda_1$ copies $O_j/r$ from $O_j$ over $\text{link}_{\hat{r}}$ if allowed by $f_{\hat{r}}$ .
Net Effect	$r \in M[k,j]$ $r \in B[k,j]$ if $\lambda(S_k) \supseteq \lambda_q$	$O_j/\hat{r} \in \text{dom}(S_k \lambda_1)$ for all $\lambda_1$ $O_j/r \in \text{dom}(S_k \lambda_1)$ if $\lambda_1 \supseteq \lambda_q$
3	$S_i$ enters $w$ in $M[k,j]$	$S_i \lambda_p$ copies $O_j/\hat{w}$ over $\text{link}_u$ to all cohorts of $S_k$ . Each cohort $S_k \lambda_1$ copies $O_j/w$ from $O_j$ over $\text{link}_{\hat{w}}$ if allowed by $f_{\hat{w}}$ .
Net Effect	$w \in M[k,j]$ $w \in B[k,j]$ if $\lambda(S_k) \supseteq \lambda_q$	$O_j/\hat{w} \in \text{dom}(S_k \lambda_1)$ for all $\lambda_1$ $O_j/w \in \text{dom}(S_k \lambda_1)$ if $\lambda_1 \supseteq \lambda_q$
4	$S_i$ changes $\lambda(S_i)$	Ignored.
Net Effect	Row $i$ of $B$ is changed to preserve simple-security and the star-property.	Accounted for in simulating operations 2 and 3.

Table 1: Simulation of BLP Operations in SPM

Operation	SPM System	BLP Simulation
1	$S_i \lambda_p$ creates $O_j$ of type $o \lambda_m$	$S_i$ creates $O_j$ with $\lambda(O_j) = \lambda_m$
Net Effect	$O_j / o \in \text{dom}(S_i \lambda_p)$ $\text{dom}(O_j) = O_j / rwr\hat{w}c$	$o \in M[i,j]$
2	$S_i \lambda_p$ copies $O_j / o : c$ from itself to its cohort $S_i \lambda_q$ over $\text{link}_k$	Ignored
Net Effect	$O_j / o : c \in \text{dom}(S_i \lambda_q)$	Accounted for in operation 1
3	$S_i \lambda_p$ copies $O_j / \hat{r} : c$ from $O_j$ to itself over $\text{link}_o$	$S_i$ enters $r$ in $M[i,j]$
Net Effect	$O_j / \hat{r} : c \in \text{dom}(S_i \lambda_p)$	$r \in M[i,j]; r \in B[i,j]$ if $\lambda(S_i) \supseteq \lambda(O_j)$
4	$S_i \lambda_p$ copies $O_j / \hat{w} : c$ from $O_j$ to itself over $\text{link}_o$	$S_i$ enters $w$ in $M[i,j]$
Net Effect	$O_j / \hat{w} : c \in \text{dom}(S_i \lambda_p)$	$w \in M[i,j]; w \in B[i,j]$ if $\lambda(S_i) \sqsubseteq \lambda(O_j)$
5	$S_i \lambda_p$ copies $O_j / \hat{r}$ from itself to $S_k \lambda_q$ over $\text{link}_u$	$S_i$ enters $r$ in $M[k,j]$
Net Effect	$O_j / \hat{r} \in \text{dom}(S_k \lambda_q)$	$r \in M[k,j]; r \in B[k,j]$ if $\lambda(S_k) \supseteq \lambda(O_j)$
6	$S_i \lambda_p$ copies $O_j / \hat{w}$ from itself to $S_k \lambda_q$ over $\text{link}_u$	$S_i$ enters $w$ in $M[k,j]$
Net Effect	$O_j / \hat{w} \in \text{dom}(S_k \lambda_q)$	$w \in M[k,j]; w \in B[k,j]$ if $\lambda(S_k) \sqsubseteq \lambda(O_j)$
7	$S_i \lambda_p$ copies $O_j / r$ from $O_j$ to itself over $\text{link}_{\hat{r}}$	Ignored
Net Effect	$O_j / r \in \text{dom}(S_i \lambda_p)$	Accounted for in operations 3 and 5
8	$S_i \lambda_p$ copies $O_j / w$ from $O_j$ to itself over $\text{link}_{\hat{w}}$	Ignored
Net Effect	$O_j / w \in \text{dom}(S_i \lambda_p)$	Accounted for in operations 4 and 6

Table 2: Simulation of SPM Operations in BLP

The correspondence between the BLP system and the SPM system by the above SPM system is almost seen in a natural and intuitive manner with the BLP system. This system is an instance of SPM not merely in a formal sense but also in an intuitively meaningful manner.

**Theorem 1** *The BLP system of definition 1 and the SPM system of definition 2 with the same initial state are equivalent.*

**Proof:** To establish equivalence between the BLP system and the SPM system, we show that every operation in the BLP system, which have read, write and owner rights. There are 8 operations in table 1 along with the SPM operations. We show that any operation executed by the BLP system is simulated by a sequence of operations initiated by the SPM system. The operation by subject  $S_i$  is simulated by letting  $S_i$  give rights to the other cohorts of  $S_i$  using  $\text{link}_k$ 's. The operations for constructing the initial state and are stated in table 1. So it is exactly the SPM cohorts of  $S_i$  which are simulated by BLP. BLP implies the right to give read or write rights. This is simulated by letting each SPM cohort obtain rights in the same manner each SPM cohort of  $S_i$  has the right to give access for its owned objects to other subjects. The right to give access to  $O_j$  is simulated in SPM by letting  $S_i$  give rights to every SPM cohort of  $S_k$ . Cohorts of  $S_k$  which are denied this ability by  $f_{\hat{r}}$ . So simple security is enforced by  $f_{\hat{r}}$ . The BLP operation of  $S_i$  giving rights to  $S_k$  in SPM with the star-property enforced by  $f_{\hat{r}}$ . Changing its current level is ignored in SPM. The operation of modifying row  $i$  of the current access matrix is ignored in the star-property. This is accounted for in SPM by the SPM cohorts of  $S_i$  at different levels.

To complete the demonstration of equivalence, we show that every SPM property that each SPM control operation in the BLP system having the same net effect. There are eight operations enumerated in table 2. Of these only operations 3, 4, 5, 6, 7 and 8 respectively convert  $\hat{r}$  and  $\hat{w}$  rights to  $r$  and  $w$  respectively by the current access matrix  $B$  which is updated by the security and the star-property whenever the access matrix changes.

Next we consider a version of BLP system which is more powerful than allowed by the simple security and the star-property.

The correspondence between the BLP model of definition 6 and its realization by the above SPM system is almost self-evident. Scheme 2 expresses the BLP rules in a natural and intuitive manner without straining the SPM notation. Thus BLP is an instance of SPM not merely in some obscure theoretical sense, but actually in an intuitively meaningful manner. We now establish the following result.

**Theorem 1** *The BLP system of definition 6 and the SPM scheme 2 with its specified initial state are equivalent.*

**Proof:** To establish equivalence between the two systems we first show how each control operation in the BLP system is simulated by a sequence of control operations in the SPM system, which have the same net effect regarding distribution of read, write and owner rights. There are four control operations in BLP shown in table 1 along with the SPM operations which simulate these. The general idea is that any operation executed by the BLP subject  $S_i$  with  $\lambda(S_i)=\lambda_p$  is simulated by a sequence of operations initiated by the SPM cohort  $S_i\lambda_p$ . Creation of object  $O_j$  by subject  $S_i$  is simulated by letting  $S_i\lambda_p$  create  $O_j$ . Ownership of  $O_j$  is transferred to the other cohorts of  $S_i$  using  $link_k$ 's. These cohort links are established in constructing the initial state and are static, since the cohort right  $k$  cannot be copied. So it is exactly the SPM cohorts of  $S_i$  who can share ownership of  $O_j$ . Ownership in BLP implies the right to give read or write access to other subjects. This is simulated by letting each SPM cohort obtain  $O_j/\hat{r}\hat{w}c$  from  $dom(O_j)$  using  $link_o$ . In this manner each SPM cohort of  $S_i$  has the authorization to give read and write access for its owned objects to other subjects. The BLP operation of  $S_i$  giving  $S_k$  read access to  $O_j$  is simulated in SPM by letting  $S_i\lambda_p$  copy  $O_j/\hat{r}$  from its own domain to every SPM cohort of  $S_k$ . Cohorts of  $S_k$  whose level dominates  $\lambda(O_j)$  can then copy  $O_j/r$  from  $dom(O_j)$  using  $link_{\hat{r}}$ . Cohorts of  $S_k$  whose level does not dominate  $\lambda(O_j)$  are denied this ability by  $f_{\hat{r}}$ . So simple security is enforced in the SPM simulation by  $f_{\hat{r}}$ . The BLP operation of  $S_i$  giving  $S_k$  write access to  $O_j$  is similarly simulated in SPM with the star-property enforced by  $f_{\hat{w}}$ . The BLP operation of subject  $S_i$  changing its current level is ignored in SPM. In BLP this operation has the effect of modifying row  $i$  of the current access matrix  $B$  to preserve simple security and the star-property. This is accounted for in the SPM simulation by the manner in which the SPM cohorts of  $S_i$  at different levels obtain read and write tickets.

To complete the demonstration of equivalence it remains to show the converse property that each SPM control operation can be simulated by a BLP operation having the same net effect. There are eight control operations in the SPM system as enumerated in table 2. Of these only operations 1, 3, 4, 5 and 6, i.e., object creation and copying of  $\hat{r}$  and  $\hat{w}$ , are explicitly simulated in BLP; respectively as object creation and giving of read and write access. SPM operation 2 establishes shared ownership of an object between SPM cohorts and needs no simulation. Operation 7 and 8 respectively convert  $\hat{r}$  and  $\hat{w}$  rights to  $r$  and  $w$ . These are accounted for in BLP by the current access matrix  $B$  which is automatically changed to preserve simple security and the star-property whenever the current security level of a subject changes.  $\square$

Next we consider a version of BLP with subjects who are given more power than allowed by the simple security and star properties of definition 6. This model

	BLP Simulation
	$S_i$ creates $O_j$ with $\lambda(O_j)=\lambda_m$ $o \in M[i,j]$
its	Ignored  Accounted for in operation 1
self	$S_i$ enters $r$ in $M[i,j]$  $r \in M[i,j]; r \in B[i,j]$ if $\lambda(S_i) \supseteq \lambda(O_j)$
self	$S_i$ enters $w$ in $M[i,j]$  $w \in M[i,j]; w \in B[i,j]$ if $\lambda(S_i) \sqsubseteq \lambda(O_j)$
$\lambda_q$	$S_i$ enters $r$ in $M[k,j]$  $r \in M[k,j]; r \in B[k,j]$ if $\lambda(S_k) \supseteq \lambda(O_j)$
$\lambda_q$	$S_i$ enters $w$ in $M[k,j]$  $w \in M[k,j]; w \in B[k,j]$ if $\lambda(S_k) \sqsubseteq \lambda(O_j)$
self	Ignored  Accounted for in operations 3 and 5
self	Ignored  Accounted for in operations 4 and 6

SPM Operations in BLP

is based on the security properties of [4] in the so-called network interpretation of multilevel security. The basic idea is to allow subjects to violate simple security and star properties in a controlled manner. This is achieved by associating a pair of security levels  $\lambda_{vmax}(S_i)$  and  $\lambda_{amin}(S_i)$  with each subject  $S_i$ , with the subscripts respectively read as view-maximum and alter-minimum. It is required that  $\lambda_{vmax}(S_i) \supseteq \lambda_{amin}(S_i)$ . The range of a subject is the set of levels bounded by  $\lambda_{vmax}$  and  $\lambda_{amin}$  as follows.

$$range(S_i) = \{\lambda_k \mid \lambda_{vmax}(S_i) \supseteq \lambda_k \supseteq \lambda_{amin}(S_i)\}$$

Subject  $S_i$  is allowed to read and write objects whose security levels are in  $range(S_i)$ . Outside this range requirements similar to simple security and the star-property are stipulated as follows.

1.  $S_i$  can read  $O_j$  only if  $\lambda_{amin}(S_i) \supseteq \lambda(O_j)$ .
2.  $S_i$  can write  $O_j$  only if  $\lambda(O_j) \supseteq \lambda_{vmax}(S_i)$ .

Since  $\lambda_{vmax} \supseteq \lambda_{amin}$  we can view these requirements as respectively generalizing simple security to require that all levels in  $range(S_i)$  dominate  $\lambda(O_j)$ , and the star-property to require that  $\lambda(O_j)$  dominates all levels in  $range(S_i)$ . Within  $range(S_i)$  simple security and the star-property are not enforced.

The SPM simulation in this case is actually simpler than the previous one since there is no notion of a changing current security level for subjects. So, there is no need for SPM cohorts. Each BLP subject is mapped to a single SPM subject whose type is determined by  $\lambda_{vmax}$  and  $\lambda_{amin}$  of the BLP subject. We have the following scheme.

### Scheme 3 BLP network interpretation with tranquility.

1.  $TS = \{\sigma\lambda_i\lambda_j \mid \lambda_i \supseteq \lambda_j\} \cup \{o\lambda_k \mid \lambda_k \in \Lambda\}$ ,  $TO = \phi$   
SPM subjects of type  $\sigma\lambda_i\lambda_j$  model BLP subjects with  $\lambda_{vmax} = \lambda_i$  and  $\lambda_{amin} = \lambda_j$ . SPM subjects of type  $o\lambda_i$  simulate BLP objects with current level  $\lambda_i$ .
2.  $RI = \{r:c, w:c\}$ ,  $RC = \{o:c, \hat{r}:c, \hat{w}:c\}$   
 $r, w$ , and  $o$  are the original BLP rights;  $\hat{r}$  and  $\hat{w}$  are artifacts of the simulation
3.  $link_u(U, V) \equiv true$   
 $link_o(U, V) \equiv U/o \in dom(V)$   
 $link_{\hat{r}}(U, V) \equiv U/\hat{r} \in dom(V)$   
 $link_{\hat{w}}(U, V) \equiv U/\hat{w} \in dom(V)$

The subscripts on these links have the following mnemonic significance:  $u$  for universal,  $o$  for owner,  $\hat{r}$  for discretionary read access, and  $\hat{w}$  for discretionary write access.

4. Undefined values of the filter functions are assumed by default to be  $\phi$ .

$$f_u(\sigma\lambda_i\lambda_j, \sigma\lambda_k\lambda_l) = \{o\lambda_m/\hat{r}\hat{w} \mid \lambda_m \in \Lambda\}$$

$$f_o(o\lambda_i, \sigma\lambda_j\lambda_k) = o\lambda_i/\hat{r}\hat{w}c$$

- $$f_{\hat{r}}(o\lambda_i, \sigma\lambda_j\lambda_k) = \text{if } \lambda_j \supseteq \lambda_i \supseteq \lambda_k$$
- $$f_{\hat{w}}(o\lambda_i, \sigma\lambda_j\lambda_k) = \text{if } \lambda_j \supseteq \lambda_i \supseteq \lambda_k$$
5.  $cc(\sigma\lambda_i\lambda_j) = \{o\lambda_k \mid \lambda_k \in \Lambda\}$
  6.  $cr_p(\sigma\lambda_i\lambda_j, o\lambda_k) = child/oc$ ,  
 $cr_c(\sigma\lambda_i\lambda_j, o\lambda_k) = child/rw\hat{r}\hat{w}c$

The initial state for the SPM system

1.  $SUB = \Omega \cup \{S_i\lambda_j\lambda_k \mid S_i \in \Sigma \wedge \lambda_j, \lambda_k \in \Lambda\}$
2.  $type(O_j) = o\lambda_1$ , where  $\lambda(O_j) = \lambda_1$   
 $type(S_i\lambda_j\lambda_k) = \sigma\lambda_j\lambda_k$
3.  $dom(O_i) = O_i/rw\hat{r}\hat{w}c$

$$dom(S_i\lambda_j\lambda_k) = \{O_1/o\hat{r}\hat{w}c \mid o \in M\}$$

$$\{O_1/\hat{r} \mid r \in M[i, j]\}$$

$$\{O_1/r \mid r \in B[i, j]\}$$

We can establish equivalence between theorem 1 for the construction of sch proof will be simpler in this case.

Next consider the Biba integrity r the aim of controlling unauthorized r thORIZED disclosure. Its definition i following replacements.

$$\supseteq$$

$$\supseteq$$

$$\lambda_{max}$$

The lattice of security levels is replace has a minimum integrity level design the subject must dominate  $\lambda_{min}$  at al are replaced by the following.

$$r \in B[i, j] \Leftrightarrow r \in M[i, j] \wedge \lambda_i \supseteq \lambda_j$$

$$w \in B[i, j] \Leftrightarrow w \in M[i, j] \wedge \lambda_i \supseteq \lambda_j$$

That is a subject is only allowed to r itself and to write objects of the sam for BLP it is no surprise that the E simply need to reverse the dominance by reversing the dominance relation o initial state. Lee [20] and Schockley [3 of the BLP network interpretation. T scheme 3. It is also possible for the

the so-called network interpretation  
 low subjects to violate simple secu-  
 r. This is achieved by associating a  
 with each subject  $S_i$ , with the sub-  
 lter-minimum. It is required that  
 the set of levels bounded by  $\lambda_{vmax}$

$$\lambda_k \supseteq \lambda_{amin}(S_i)$$

whose security levels are in  $range(S_i)$ .  
 simple security and the star-property

ements as respectively generalizing  
 $range(S_i)$  dominate  $\lambda(O_j)$ , and the star-  
 levels in  $range(S_i)$ . Within  $range(S_i)$   
 enforced.

simpler than the previous one since  
 rity level for subjects. So, there is  
 is mapped to a single SPM subject  
 of the BLP subject. We have the

tranquility.

$$D = \phi$$

subjects with  $\lambda_{vmax} = \lambda_i$  and  $\lambda_{amin} =$   
 LP objects with current level  $\lambda_i$ .

and  $\hat{w}$  are artifacts of the simulation

following mnemonic significance:  $u$  for  
 read access, and  $\hat{w}$  for discretionary

are assumed by default to be  $\phi$ .

$$f_{\hat{r}}(o\lambda_i, \sigma\lambda_j\lambda_k) = \text{if } \lambda_j \supseteq \lambda_i \supseteq \lambda_k \vee \lambda_k \supseteq \lambda_i \text{ then } o\lambda_i/r \text{ else } \phi$$

$$f_{\hat{w}}(o\lambda_i, \sigma\lambda_j\lambda_k) = \text{if } \lambda_j \supseteq \lambda_i \supseteq \lambda_k \vee \lambda_i \supseteq \lambda_j \text{ then } o\lambda_i/w \text{ else } \phi$$

$$5. cc(\sigma\lambda_i\lambda_j) = \{o\lambda_k \mid \lambda_k \in \Lambda\}$$

$$6. cr_p(\sigma\lambda_i\lambda_j, o\lambda_j) = child/oc,$$

$$cr_c(\sigma\lambda_i\lambda_j, o\lambda_j) = child/rw\hat{r}\hat{w}c$$

The initial state for the SPM system is as follows.

$$1. SUB = \Omega \cup \{S_i\lambda_j\lambda_k \mid S_i \in \Sigma \wedge \lambda_{vmax}(S_i) = \lambda_j \wedge \lambda_{amin}(S_i) = \lambda_k\}$$

$$2. type(O_j) = o\lambda_i, \text{ where } \lambda(O_j) = \lambda_i$$

$$type(S_i\lambda_j\lambda_k) = \sigma\lambda_j\lambda_k$$

$$3. dom(O_i) = O_i/rw\hat{r}\hat{w}c$$

$$dom(S_i\lambda_j\lambda_k) = \{O_i/o\hat{r}\hat{w}c \mid o \in M[i,1]\} \cup$$

$$\{O_i/\hat{r} \mid r \in M[i,1]\} \cup \{O_i/\hat{w} \mid w \in M[i,1]\} \cup$$

$$\{O_i/r \mid r \in B[i,1]\} \cup \{O_i/w \mid w \in B[i,1]\}$$

We can establish equivalence between the BLP and SPM systems as was done in  
 theorem 1 for the construction of scheme 2. Because of the absence of cohorts the  
 proof will be simpler in this case.

Next consider the Biba integrity model [6] which is the exact dual of BLP with  
 the aim of controlling unauthorized modification of information rather than unau-  
 thorized disclosure. Its definition is obtained from definition 6 by making the  
 following replacements.

$$\supseteq \leftrightarrow \sqsubseteq$$

$$\supset \leftrightarrow \sqsubset$$

$$\lambda_{max} \leftrightarrow \lambda_{min}$$

The lattice of security levels is replaced by a lattice of integrity levels. Each subject  
 has a minimum integrity level designated by  $\lambda_{min}$ . The current integrity level of  
 the subject must dominate  $\lambda_{min}$  at all times. Simple security and the star-property  
 are replaced by the following.

$$r \in B[i,j] \Leftrightarrow r \in M[i,j] \wedge \lambda(S_i) \sqsubseteq \lambda(O_j) \quad \text{Simple integrity}$$

$$w \in B[i,j] \Leftrightarrow w \in M[i,j] \wedge \lambda(S_i) \supseteq \lambda(O_j) \quad \text{Integrity star-property}$$

That is a subject is only allowed to read objects of the same or higher integrity as  
 itself and to write objects of the same or lower integrity. Given the constructions  
 for BLP it is no surprise that the Biba model can be instantiated in SPM. We  
 simply need to reverse the dominance relations in  $f_{\hat{r}}$  and  $f_{\hat{w}}$  of scheme 2. Similarly  
 by reversing the dominance relation of the BLP construction we obtain the proper  
 initial state. Lee [20] and Schockley [39] formulate integrity models which are duals  
 of the BLP network interpretation. These can be expressed in SPM as the dual of  
 scheme 3. It is also possible for the Biba and BLP models to coexist in a single

system. If the same lattice is used for both models, their coexistence implies that a subject can read or write only at its current level. More generally the two models can coexist with independent lattices, so each subject and object has a security level and an independent integrity level. Such coexistence can be easily modeled in SPM by combining the rules of the two models. We conclude that the mandatory controls of the BLP model for non-disclosure and of the Biba model for integrity are special cases of the more general mandatory controls of SPM.

Finally it is worth considering what kind of non-tranquility can be accommodated in a monotonic manner in SPM. To be specific consider scheme 2. Non-tranquility in BLP is usually specified by including a security officer subject who has the authority to enroll new subjects, change  $\lambda_{\max}$  of existing subjects, and change  $\lambda$  of existing objects. In SPM we can define a new subject type *sec-off* with one instance in the initial state to model the security officer. Creation of new subjects can be simulated by allowing the security officer to create a collection of SPM cohorts and giving him the ability to connect them by  $\text{link}_k$ 's. Changing  $\lambda_{\max}(S_i)$  from  $\lambda_p$  to  $\lambda_q$  can be similarly modeled so long as  $\lambda_q \supseteq \lambda_p$ . The security officer simply introduces new cohorts of  $S_i$  at levels dominated by  $\lambda_q$  but not by  $\lambda_p$ , and connects these to the existing cohorts of  $S_i$  and each other by  $\text{link}_k$ 's. If  $\lambda_q \not\supseteq \lambda_p$  we need to delete some of the existing cohorts. Since this might delete some cohorts which existed in the initial state we would need to treat this as being a different SPM system. Changing the security level of an object requires revocation of read and write privileges to preserve simple security and the star-property, and would again have to be treated as a transition to a different SPM system.\* Note that with this kind of unrestricted power given to a security officer there really is no safety in the system, unless we assume the security officer does not change security levels arbitrarily. So for purpose of safety analysis one does assume some form of tranquility.

## 6 Take-Grant Models

Of all the models discussed in this paper, take-grant is closest in viewpoint to SPM. Its SPM simulation is, therefore, a very natural one. Take-grant derives its name from its two control rights *t* (take) and *g* (grant). Several papers have been published on this model, including [16, 21, 40]. Inevitably there are slight differences in the precise definition of the model in these papers. Our presentation follows Snyder's review [40] of the model most closely. Several variations of take-grant have also been proposed [9, 23]. These variations are also easily specified in SPM.

The SPM simulation of the basic take-grant model is given as schemes VI through VIII of [34]. In this paper we extend the construction to accommodate analysis of theft in take-grant [41]. This shows how assumptions about behavior are easily expressed in SPM.

Transfer of information in the take-grant model has been analyzed by Bishop and Snyder [7]. The control operations used for this purpose are a special case of grammatical protection systems which are modeled in SPM in the next Section. Analysis of combined authority and information transfer and theft [8] can be accommodated in SPM by combining the constructions of this Section with those

of the following Section. In this way take-grant within the analysis framev

Let us briefly review scheme VIII subject-object version of take-grant subjects: *as* for active subjects and cannot execute operations and is me the take-grant model defines two link have obvious mnemonic significance.

$$\begin{aligned} \text{link}_g(U, V) \\ \text{link}_t(U, V) \end{aligned}$$

A  $\text{link}_g$  requires a grant capability capability at the destination. A link  $c$  in the domain of an active subject,  $i$  active whereas  $\text{link}_t(U, V)$  can be exer no selectivity in the copy operation. subjects whereas active subjects can this is easily specified in SPM as follo

**Scheme 4** The take-grant model with

1.  $TS = \{as, ps\}$ ,  $TO = \phi$
2.  $RC = \{t:c, g:c\}$ ,  $RI = \text{some fini}$
3.  $\text{link}_g(U, V) \equiv V/g \in \text{dom}(U)$   
 $\text{link}_t(U, V) \equiv U/t \in \text{dom}(V)$
4.  $f_g(as, [as|ps]) = T \times R$   
 $f_g(ps, [as|ps]) = \phi$   
 $f_t([as|ps], as) = T \times R$   
 $f_t([as|ps], ps) = \phi$
5.  $cc(as) = \{as, ps\}$   
 $cc(ps) = \phi$
6. There is a uniform create-rule w

Here we introduce abbreviated notatio tion of  $[as|ps]$  is that it is an abbrev terms. For example in the above case be as follows.

\*\*The equivalence is not absolute since, str a subject to possess tickets for itself. It ap without some drastic step such as declaring e However, as observed by Snyder [40] this is the restriction is unnecessarily restrictive in itself so as to give these to other subjects at that all tickets in the SPM initial state are c



ls, their coexistence implies that a el. More generally the two models subject and object has a security existence can be easily modeled in We conclude that the mandatory d of the Biba model for integrity controls of SPM.

non-tranquility can be accommo- specific consider scheme 2. Non- ing a security officer subject who ge  $\lambda_{max}$  of existing subjects, and define a new subject type *sec-off* e security officer. Creation of new ty officer to create a collection of nect them by  $link_x$ 's. Changing d so long as  $\lambda_q \supseteq \lambda_p$ . The security vels dominated by  $\lambda_q$  but not by f  $S_i$  and each other by  $link_x$ 's. If orts. Since this might delete some ould need to treat this as being a el of an object requires revocation ecurity and the star-property, and to a different SPM system.\* Note to a security officer there really e security officer does not change ty analysis one does assume some

ke-grant is closest in viewpoint to natural one. Take-grant derives d g (grant). Several papers have t, 40]. Inevitably there are slight in these papers. Our presentation closely. Several variations of take- riations are also easily specified in

at model is given as schemes VI the construction to accommodate how assumptions about behavior

odel has been analyzed by Bishop this purpose are a special case of eled in SPM in the next Section. n transfer and theft [8] can be ac- onctions of this Section with those

of the following Section. In this way we are able to cover the analysis results of take-grant within the analysis framework of Section 3 for SPM.

Let us briefly review scheme VIII of [34] which is equivalent\*\* to the so called subject-object version of take-grant [16]. In this scheme there are two types of subjects: *as* for active subjects and *ps* for passive subjects. A passive subject cannot execute operations and is merely a repository for tickets. In SPM terms, the take-grant model defines two link predicates as follows, where the subscripts have obvious mnemonic significance.

$$\begin{aligned} link_g(U,V) &\equiv V/g \in \text{dom}(U) \\ link_t(U,V) &\equiv U/t \in \text{dom}(V) \end{aligned}$$

A  $link_g$  requires a grant capability at the source while a  $link_t$  requires a take capability at the destination. A link can be exercised only if authorized by a ticket in the domain of an active subject, i.e.,  $link_g(U,V)$  can be exercised only if  $U$  is active whereas  $link_t(U,V)$  can be exercised only if  $V$  is active. There is otherwise no selectivity in the copy operation. Passive subjects are not allowed to create subjects whereas active subjects can create both passive and active subjects. All this is easily specified in SPM as follows.

**Scheme 4** *The take-grant model with passive subjects.*

1.  $TS = \{as, ps\}$ ,  $TO = \phi$
2.  $RC = \{t:c, g:c\}$ ,  $RI =$  some finite set disjoint from  $RC$
3.  $link_g(U,V) \equiv V/g \in \text{dom}(U)$   
 $link_t(U,V) \equiv U/t \in \text{dom}(V)$
4.  $f_g(as, [as|ps]) = T \times R$   
 $f_g(ps, [as|ps]) = \phi$   
 $f_t([as|ps], as) = T \times R$   
 $f_t([as|ps], ps) = \phi$
5.  $cc(as) = \{as, ps\}$   
 $cc(ps) = \phi$
6. There is a uniform create-rule with  $cr_p = child/R$  and  $cr_c = \phi$

Here we introduce abbreviated notation to keep the scheme compact. The interpretation of  $[as|ps]$  is that it is an abbreviation for all combinations of the bracketed terms. For example in the above case the verbose definition of  $f_g$  is understood to be as follows.

\*\*The equivalence is not absolute since, strictly speaking, the take-grant model does not allow a subject to possess tickets for itself. It appears this restriction cannot be specified in SPM, without some drastic step such as declaring each subject to be of a distinct type unique to itself. However, as observed by Snyder [40] this is not a fundamental feature of take-grant. Moreover, the restriction is unnecessarily restrictive in that we often want a subject to possess rights for itself so as to give these to other subjects at that subject's discretion. We also need to assume that all tickets in the SPM initial state are copiable.

$$\begin{aligned}
 f_g(as, as) &= T \times R \\
 f_g(as, ps) &= T \times R \\
 f_g(ps, as) &= \phi \\
 f_g(ps, ps) &= \phi
 \end{aligned}$$

In addition to its compactness this notation is useful in highlighting the similarities and differences between types.

The create-rule in scheme 4 is not attenuating and there is a loop in  $cc$  due to  $as \in cc(as)$ . So as it stands the scheme is not acyclic attenuating and thereby does not fall within the known decidable cases of SPM. However, scheme 4 can easily be modified to be attenuating by the technique described in [34] of distinguishing the initial set of subjects from those created subsequently. Scheme 5 also can be made non-attenuating in the same way.

We now show how the notion of theft as defined by Snyder [41] for the take-grant model can be specified by an SPM scheme. This notion assumes that certain subjects will not carry out particular operations even though they are authorized to do so. That is these subjects are trusted not to cooperate in some specific way for propagating tickets. There are numerous assumptions about behavior that one could make. Snyder analyzes a particular set of assumptions, but would need to carry out similar and perhaps more complicated analysis if these assumptions are changed. One of the great advantages of SPM is that assumptions about the behavior of subjects can be easily specified as part of a scheme. To demonstrate this we show how the specific assumptions used by Snyder are stated in SPM.

Snyder's concept of theft is that a ticket  $Y/x$  is stolen by a subject  $U$  provided the following conditions hold.

1.  $U$  does not possess  $Y/x$  in the initial state.
2. Subjects who possess  $Y/x$  in the initial state do not grant  $Y/x$  to any other subject, i.e., subjects possessing  $Y/x$  are trusted not to give it away.
3. There is a reachable state with  $Y/x \in \text{dom}(U)$ .

In other words theft is said to occur if  $U$  is able to obtain  $Y/x$ , even if subjects possessing  $Y/x$  in the initial state do not give it away to anybody. We can model these assumptions in SPM by distinguishing different types of subjects. First we distinguish trusted subjects from untrusted ones. Since passive subjects cannot exercise the grant operation, this distinction applies only to active subjects. Next we need to distinguish entities that are confidential from those that are non-confidential. The assumed behavior of trusted subjects applies only to confidential entities. They are free to grant tickets for non-confidential entities, but are constrained by their behavior in granting tickets for confidential entities. The notions of trusted and confidential are independent attributes of subjects, so we need to define subjects types for all possible combinations of these as given in the top four rows of table 3. Passive subjects are unable to exercise the grant privilege, so they are inherently trusted. This gives us the two bottom rows of table 3.

Finally we identify the rights  $RT$  which will not be granted by trusted subjects for confidential entities. That is trusted subjects are assumed not to grant tickets

TYPE	TRUSTED
<i>tcas</i>	Yes
<i>tnas</i>	Yes
<i>ucas</i>	No
<i>unas</i>	No
<i>cps</i>	—
<i>nps</i>	—

Table 3: SPM Subject Types

of type  $\{tcas, ucas, cps\} \times RT$  even if all subjects are of type  $\{tcas, ucas, cps\} \times (R-RT)$ . This is specified by the create-rule for subjects to all other subjects to be  $f_g$  from untrusted subjects to all other subjects. The values of  $f_t$  remain unchanged.

This results in the following scheme

**Scheme 5** *Theft of rights in the take-grant model*

1.  $TS = \{tcas, tnas, ucas, unas, cps, nps\}$   
Let  $CS = \{tcas, ucas, cps\}$  be the set of confidential entities
2.  $RC = \{t:c, g:c\}$ ,  $RI = \text{some finite set}$   
Let  $RT \subseteq R$  be the set of rights
3.  $\text{link}_g(U, V) \equiv V/g \in \text{dom}(U)$   
 $\text{link}_t(U, V) \equiv U/t \in \text{dom}(V)$
4. Let  $[TS] \equiv [tcas|tnas|ucas|unas|ucas|unas|cps|nps]$   
 $f_g([tcas|tnas], [TS]) = (T \times R) \times RT$   
 $f_g([ucas|unas], [TS]) = T \times R$   
 $f_g([cps|nps], [TS]) = \phi$   
 $f_t([TS], [tcas|tnas|ucas|unas]) = \phi$   
 $f_t([TS], [cps|nps]) = \phi$
5.  $cc([tcas|tnas|ucas|unas]) = \phi$   
 $cc([cps|nps]) = \phi$
6. There is a uniform create-rule with

We can as easily model a different set of assumptions if subjects do not grant any take rights. This can be done simply need to change  $f_g$  as follows.

$$f_g([tcas|tnas], [TS]) = \phi$$

TYPE	TRUSTED	CONFIDENTIAL	ACTIVE
<i>tcas</i>	Yes	Yes	Yes
<i>tnas</i>	Yes	No	Yes
<i>ucas</i>	No	Yes	Yes
<i>unas</i>	No	No	Yes
<i>cps</i>	—	Yes	No
<i>nps</i>	—	No	No

Table 3: SPM Subject Types for Modeling Theft in Take-Grant

of type  $\{tcas, ucas, cps\} \times RT$  even if authorized to so, but may grant tickets of type  $\{tcas, ucas, cps\} \times (R - RT)$ . This is specified by setting the value of  $f_g$  from trusted subjects to all other subjects to be  $(T \times R) - (\{tcas, ucas, cps\} \times RT)$ . The value of  $f_g$  from untrusted subjects to all other subjects remains unchanged as  $T \times R$ . All values of  $f_t$  remain unchanged.

This results in the following scheme.

**Scheme 5** *Theft of rights in the take-grant model.*

1.  $TS = \{tcas, tnas, ucas, unas, cps, nps\}$ ,  $TO = \phi$   
Let  $CS = \{tcas, ucas, cps\}$  be the set of confidential types
2.  $RC = \{t:c, g:c\}$ ,  $RI =$  some finite set disjoint from  $RC$   
Let  $RT \subseteq R$  be the set of rights whose theft we are analyzing
3.  $link_g(U, V) \equiv V/g \in \text{dom}(U)$   
 $link_t(U, V) \equiv U/t \in \text{dom}(V)$
4. Let  $[TS] \equiv [tcas|tnas|ucas|unas|cps|nps]$   
 $f_g([tcas|tnas], [TS]) = (T \times R) - (CS \times RT)$   
 $f_g([ucas|unas], [TS]) = T \times R$   
 $f_g([cps|nps], [TS]) = \phi$   
 $f_t([TS], [tcas|tnas|ucas|unas]) = T \times R$   
 $f_t([TS], [cps|nps]) = \phi$
5.  $cc([tcas|tnas|ucas|unas]) = \{unas, nps\}$   
 $cc([cps|nps]) = \phi$
6. There is a uniform create-rule with  $cr_p = \text{child}/R$  and  $cr_c = \phi$

We can as easily model a different notion of theft in which say the trusted subjects do not grant any take rights in addition to the above restriction. We simply need to change  $f_g$  as follows.

$$f_g([tcas|tnas], [TS]) = (T \times R) - (CS \times RT \cup TS \times \{t:c\})$$

Or perhaps the assumption that trusted subjects do not grant any take rights except to other trusted subjects, specified by modifying  $f_g$  as follows.

$$\begin{aligned} f_g([t_{cas}|t_{nas}], [u_{cas}|u_{nas}|c_{ps}|n_{ps}]) &= (T \times R) - (CS \times RT \cup TS \times \{t:c\}) \\ f_g([t_{cas}|t_{nas}], [t_{cas}|t_{nas}]) &= (T \times R) - (CS \times RT) \end{aligned}$$

The structure of SPM gives us a powerful framework for investigating the consequences of such assumptions about behavior. In the take-grant framework each of these separate notions of theft would require a separate analysis along the lines of [7, 8, 16, 21, 41]. In SPM these alternate notions require separate schemes, but the same analysis algorithm of Section 3 can be used in all cases. Moreover, in the SPM framework ad hoc assumptions about behavior can be accommodated quite easily. In the limit each individual user may be treated separately for this purpose.

## 7 Grammatical Protection Systems

Grammatical protection systems (GPS) were defined by Lipton and Budd [22] and shown to have a close relation to context-free grammars. Safety in these systems is reduced to a parsing problem which is decidable in polynomial time. The subsumption of GPS by SPM demonstrates the ability of SPM to simulate models whose control operations are at first sight quite contrary to SPM control operations. It is also significant because, in combination with the constructions of the previous Section, it allows us to accommodate notions of information and authority transfer and theft in take-grant within SPM.

There is no create operation in GPS so the system has a fixed set of subjects. The protection state of the system is visualized as a graph in which there is a directed edge labeled  $\alpha$  from  $U$  to  $V$ , shown as  $U \xrightarrow{\alpha} V$ , if  $U$  possesses the set of rights  $\alpha$  for  $V$ . In other words,  $U \xrightarrow{\alpha} V$  if and only if subject  $U$  possesses the tickets  $V/\alpha$ .

The rules for changing the protection state are expressed in one of the forms indicated in table 4 where  $\alpha$ ,  $\beta$  and  $\gamma$  are non-empty sets of rights. The interpretation of these rules is straightforward. A class I rule says that if  $U$  possesses  $V/\alpha$  and  $V$  possesses  $W/\beta$  then  $U$  can acquire  $W/\gamma$ . Similarly a class II rule says that if  $U$  possesses  $V/\alpha$  and  $W$  possesses  $V/\beta$  then  $U$  can acquire  $W/\gamma$ . Class III and IV rules are similarly interpreted.

The relation of these systems to context-free grammars is strongest when  $\alpha$ ,  $\beta$  and  $\gamma$  are singleton sets. However the safety analysis algorithms are applicable to the more general case where they are arbitrary non-empty sets [10]. The rules for modeling transfer of information in the take-grant model [7] are actually instances of GPS rules as shown in table 5. Here  $r$  and  $w$  are the standard read and write privileges, while  $r'$  is a pseudo-privilege denoting implicit read. So we do have a realistic interpretation for each of the rule classes. Moreover the simulation of grammatical protection systems in SPM thereby also subsumes the information transfer analysis of the take-grant model. Combined analysis of authority and information transfer and theft in take-grant can be accommodated in SPM by combining the constructions of this Section with those of the previous one.

For future reference we define grammatical protection systems as follows.

Rule Class	
I	$U \xrightarrow{\alpha}$
II	$U \xleftarrow{\alpha}$
III	$U \xrightarrow{\alpha}$
IV	$U \xleftarrow{\alpha}$

Table 4: Rules in Gram

Rule Class	Rule	
I	Spy	U
II	Post	U
III	Pass	U
IV	Find	U

Table 5: Take-Grant Inform

**Definition 7** A grammatical protection system is a tuple  $(S, R, \alpha)$  where  $S$  is a finite set of subjects,  $R$  is a fixed set of rights and  $\alpha$  is a fixed set of tickets in table 4.

The general definition of grammatical protection systems is a notion of subject types. The rules are of specific types as specified for each class. The simulation can be reduced to GPS with a single type information. The rules effectively encode the type information [10].

GPS rules appear in many ways contrary to the take-grant challenge for the expressive power of SPM. A ticket introduced by a GPS rule may not be used in applying the rule. For instance in a class I rule  $U \xrightarrow{\alpha} V$  but  $U$  ends up with  $W/\gamma$  where  $\beta$  and  $\gamma$ . In class II and IV rules there may be no tickets.

We are able to get around this problem by using a number of SPM subjects of different types. The subjects simulate the GPS subject  $X$  are said to be of type  $X$ . The general idea of using several cohorts to simulate the GPS subject  $X$  is to use of cohorts in simulating the multilevel security model. The connections between the cohorts are now quite different. In this case the cohorts explain the underlying intuition.

subjects do not grant any take rights by modifying  $f_g$  as follows.

$$(T \times R) - (CS \times RT \cup TS \times \{t:c\})$$

$$(T \times R) - (CS \times RT)$$

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or. In the take-grant framework each  
quire a separate analysis along the lines  
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thereby also subsumes the information  
. Combined analysis of authority and  
ant can be accommodated in SPM by  
n with those of the previous one.  
ical protection systems as follows.

Rule Class	Given	Add
I	$U \xrightarrow{\alpha} V \xrightarrow{\beta} W$	$U \xrightarrow{\gamma} W$
II	$U \xrightarrow{\alpha} V \xleftarrow{\beta} W$	$U \xrightarrow{\gamma} W$
III	$U \xleftarrow{\alpha} V \xrightarrow{\beta} W$	$U \xrightarrow{\gamma} W$
IV	$U \xleftarrow{\alpha} V \xleftarrow{\beta} W$	$U \xrightarrow{\gamma} W$

Table 4: Rules in Grammatical Protection Systems

Rule Class	Rule	Given	Add
I	Spy	$U \xrightarrow{r} V \xrightarrow{r} W$	$U \xrightarrow{r'} W$
II	Post	$U \xrightarrow{r} V \xleftarrow{w} W$	$U \xrightarrow{r'} W$
III	Pass	$U \xleftarrow{w} V \xrightarrow{r} W$	$U \xrightarrow{r'} W$
IV	Find	$U \xleftarrow{w} V \xleftarrow{w} W$	$U \xrightarrow{r'} W$

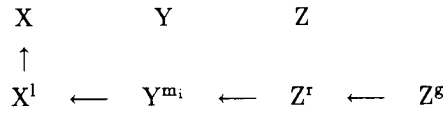
Table 5: Take-Grant Information Transfer Rules in GPS

**Definition 7** A grammatical protection system is defined by specifying a fixed set of subjects, a fixed set of rights and a fixed collection of rules of the form indicated in table 4.

The general definition of grammatical protection systems actually includes the notion of subject types. The rules are typed in that U, V and W are required to be of specific types as specified for each rule. However GPS with multiple types can be reduced to GPS with a single type by introducing new right symbols which effectively encode the type information [10]. So it suffices to consider GPS without types.

GPS rules appear in many way contrary to SPM operations and offer a significant challenge for the expressive power of SPM. The major problem is that the rights introduced by a GPS rule may not be present in any subject's domain prior to applying the rule. For instance in a class I or III rule, V is required to possess  $W/\beta$  but U ends up with  $W/\gamma$  where  $\beta$  and  $\gamma$  may not be related in any way. In class II and IV rules there may be no tickets at all for W and yet U acquires  $W/\gamma$ .

We are able to get around this problem by simulating each GPS subject by a number of SPM subjects of different types. The collection of SPM subjects which simulate the GPS subject X are said to be cohorts of X and of each other. This general idea of using several cohorts to simulate a single subject is similar to our use of cohorts in simulating the multilevel security models of Section 5. Of course the connections between the cohorts and the role they play in the simulation are now quite different. In this case the construction is much more intricate. We now explain the underlying intuition.

Figure 1: Simulation of GPS rule  $i$  in SPM

For a GPS system with  $n$  rules, numbered 1 through  $n$ , we define the following subject types in SPM.

$$TS = \{s, sg, sl, sr, sm_1, sm_2, \dots, sm_n\}$$

The intention is that a GPS subject  $X$  be simulated by a set of SPM cohorts which has one member of each of these types, as shown below.

$$\text{SPM cohorts of } X = \{X, X^s, X^l, X^r, X^{m_1}, \dots, X^{m_n}\}$$

By convention the type of each SPM cohort is  $s$  concatenated with the superscript on the cohort's name. So  $X^s$  is of type  $sg$ ,  $X^l$  of type  $sl$ , and so on. If the superscript is missing, as in  $X$ , its type is simply  $s$ . The type  $s$  SPM subjects are the ones which simulate the actual domain of GPS subjects. We will establish that  $Y/\alpha \in \text{dom}(X)$  in the SPM system if and only if  $X \xrightarrow{\alpha} Y$  in the GPS system. However  $\text{dom}(X)$  may contain tickets for subjects of type other than  $s$ , which are used for the simulation.

We regard the type  $s$  SPM cohorts to be the one which "truly" simulate each GPS subject. SPM subjects of type other than  $s$  are an artifact of the simulation. The SPM cohorts  $X^l$  and  $X^r$  respectively simulate the role of the GPS subject  $X$  when a rule with  $X$  at the left end or right end is invoked. When a rule with the GPS subject  $X$  in the middle is invoked we have a different cohort of  $X$  for each rule. The role of the GPS subject  $X$  as the middle subject in rule  $i$  is simulated by its cohort  $X^{m_i}$ . In these cases the superscripts have obvious mnemonic significance. It remains to consider SPM subjects of type  $sg$ . In our construction the cohort  $X^s$  serves as a source or generator of tickets for the GPS subject  $X$ , when a rule with  $X$  at the right end is invoked (note that in table 4 it is always the subject at the left end which acquires rights for the subject at the right end). Each generator cohort  $X^s$  possesses all tickets for itself and for  $X$ , that is  $\text{dom}(X^s)$  contains  $X^s/R$  as well as  $X/R$ .

Invocation of rule  $i$  in the GPS system with  $X$ ,  $Y$  and  $Z$  as the left, middle and right subjects respectively is simulated as depicted in figure 1 (where each directed edge denotes an SPM link). A sequence of links is established from  $Z^s$  to  $Z^r$  to  $Y^{m_i}$  to  $X^l$  to  $X$ . The SPM scheme ensures that the links from  $Z^r$  to  $Y^{m_i}$  and from  $Y^{m_i}$  to  $X^l$ , in this sequence, can be established if and only if rule  $i$  is authorized in the GPS system. Let  $X$  obtain  $Z/\gamma$  as a result of invoking rule  $i$  in the GPS system. The links and filter functions in the SPM scheme are defined so it is possible to copy exactly  $Z^s/\gamma c$  from  $\text{dom}(Z^s)$  to  $\text{dom}(X)$  using the above sequence of links. Finally by virtue of possessing  $Z^s/\gamma$ ,  $X$  is allowed to obtain  $Z/\gamma c$  from  $\text{dom}(Z^s)$ .

Let  $RG$  be the set of rights in the GPS system, extended to occur with and

without the SPM copy flag. Let  $k$  be occur in  $RG$ . We define the rights in the GPS has no copy flag, we will make sure have the SPM copy flag. We say that tickets. All other types of tickets are s

The SPM cohorts of  $X$  are connected sets up a cohort link from  $X$  to each authorized by a  $k$  ticket at the destination requiring the  $k$  ticket at the source. A by an inverse-cohort link in the opposite respectively as follows.

$$\text{link}_k(U, V) :$$

$$\text{link}_{\hat{k}}(U, V) :$$

We define  $f_k$  from  $s$  to  $sl$ ,  $sr$  and  $sm_i$  t GPS ticket in  $\text{dom}(X)$  can be copied to way that these cohorts of  $X$  can acquire  $X$  and its  $X^l$ ,  $X^r$  and  $X^{m_i}$  cohorts is th These non-GPS tickets play a crucial ro

To simulate the GPS rules we define  $\sigma$  which occurs as  $\alpha$ ,  $\beta$  or  $\gamma$  in a GPS r

$$\text{link}_\sigma(U, V) \equiv U/\sigma \in \text{dom}($$

$$\text{link}_{\hat{\sigma}}(U, V) \equiv V/\sigma \in \text{dom}($$

Now consider a class I GPS rule. I tem with  $X$ ,  $Y$  and  $Z$  respectively as t dicated in part I of table 6. The SP  $Z/\beta c \in \text{dom}(Y)$  and  $Y/\alpha c \in \text{dom}(X)$ . To  $Y^{m_i}$  using  $\text{link}_k(Y, Y^{m_i})$ . The resulting li  $Z$  to  $Y^{m_i}$ . This sets up  $\text{link}_\beta(Z^r, Y^{m_i})$ . S  $\text{link}_k(X, X^l)$ . The resulting  $\text{link}_\alpha(Y, X^l)$  ting up  $\text{link}_\alpha(Y^{m_i}, X^l)$ . So at this point For class II rules we have a similar simu establish  $\text{link}_\alpha(Y^{m_i}, X^l)$  as in the class I c that  $Y/\beta c \in \text{dom}(Z)$ . So in the simulatio  $\text{link}_k(Z, Z^r)$ , using the resulting  $\text{link}_\beta(Y$  sets up  $\text{link}_\beta(Z^r, Y^{m_i})$ . Class III and IV table 6.

The net effect, with respect to figur  $\alpha$  edge we establish  $\text{link}_\alpha(Y^{m_i}, X^l)$ , whil establish  $\text{link}_{\hat{\alpha}}(Y^{m_i}, X^l)$ . Similarly for rul  $\text{link}_\beta(Z^r, Y^{m_i})$ , while for rules with a right To ensure that these links can be establis rule is authorized we define  $f_\sigma$  from a ty

without the SPM copy flag. Let  $k$  be a symbol, denoting cohort, which does not occur in  $RG$ . We define the rights in the SPM simulation to be  $RG \cup \{k:c\}$ . Since GPS has no copy flag, we will make sure that all rights in  $RG$  which are in  $\text{dom}(X)$  have the SPM copy flag. We say that tickets of type  $s/r:c$  for  $r \in RG$  are GPS tickets. All other types of tickets are said to be non-GPS tickets.

The SPM cohorts of  $X$  are connected to  $X$  by placing  $X/k$  in their domains. This sets up a cohort link from  $X$  to each of its cohorts. Note that the cohort link is authorized by a  $k$  ticket at the destination. We also define the inverse-cohort link by requiring the  $k$  ticket at the source. A cohort link is therefore always accompanied by an inverse-cohort link in the opposite direction. The formal definitions are respectively as follows.

$$\begin{aligned} \text{link}_k(U, V) &\equiv U/k \in \text{dom}(V) \\ \text{link}_{\bar{k}}(U, V) &\equiv V/k \in \text{dom}(U) \end{aligned}$$

We define  $f_k$  from  $s$  to  $sl$ ,  $sr$  and  $sm_i$  to be  $s/RG$ . This has the effect that every GPS ticket in  $\text{dom}(X)$  can be copied to  $X^1$ ,  $X^r$  and  $X^{m_i}$ . Moreover this is the only way that these cohorts of  $X$  can acquire GPS tickets. A further connection between  $X$  and its  $X^1$ ,  $X^r$  and  $X^{m_i}$  cohorts is that  $X$  possesses all tickets for these cohorts. These non-GPS tickets play a crucial role in the simulation as explained below.

To simulate the GPS rules we define  $\sigma$  and inverse- $\sigma$  links for each set of rights  $\sigma$  which occurs as  $\alpha$ ,  $\beta$  or  $\gamma$  in a GPS rule, respectively as follows.

$$\begin{aligned} \text{link}_\sigma(U, V) &\equiv U/\sigma \in \text{dom}(V) \equiv (\forall p \in \sigma) U/p \in \text{dom}(V) \\ \text{link}_{\bar{\sigma}}(U, V) &\equiv V/\sigma \in \text{dom}(U) \equiv (\forall p \in \sigma) V/p \in \text{dom}(U) \end{aligned}$$

Now consider a class I GPS rule. Let this rule be invoked in the GPS system with  $X$ ,  $Y$  and  $Z$  respectively as the left, middle and right subjects as indicated in part I of table 6. The SPM counterpart of the given state is that  $Z/\beta c \in \text{dom}(Y)$  and  $Y/\alpha c \in \text{dom}(X)$ . To simulate the rule we copy  $Z/\beta$  from  $Y$  to  $Y^{m_i}$  using  $\text{link}_k(Y, Y^{m_i})$ . The resulting  $\text{link}_\beta(Z, Y^{m_i})$  is then used to copy  $Z^r/\beta$  from  $Z$  to  $Y^{m_i}$ . This sets up  $\text{link}_\beta(Z^r, Y^{m_i})$ . Similarly we copy  $Y/\alpha$  from  $X$  to  $X^1$  using  $\text{link}_k(X, X^1)$ . The resulting  $\text{link}_\alpha(Y, X^1)$  is used to copy  $Y^{m_i}/\alpha$  from  $Y$  to  $X^1$ , setting up  $\text{link}_\alpha(Y^{m_i}, X^1)$ . So at this point we have  $\text{link}_\beta(Z^r, Y^{m_i})$  and  $\text{link}_\alpha(Y^{m_i}, X^1)$ . For class II rules we have a similar simulation indicated in part II of table 6. We establish  $\text{link}_\alpha(Y^{m_i}, X^1)$  as in the class I case. However the given state now requires that  $Y/\beta c \in \text{dom}(Z)$ . So in the simulation we now copy  $Y/\beta$  from  $Z$  to  $Z^r$  using  $\text{link}_k(Z, Z^r)$ , using the resulting  $\text{link}_\beta(Y, Z^r)$  to copy  $Y^{m_i}/\beta$  from  $Y$  to  $Z^r$ . This sets up  $\text{link}_\beta(Z^r, Y^{m_i})$ . Class III and IV rules are similarly simulated as shown in table 6.

The net effect, with respect to figure 1, is that for rules with a left to right  $\alpha$  edge we establish  $\text{link}_\alpha(Y^{m_i}, X^1)$ , while for rules with a right to left  $\alpha$  edge we establish  $\text{link}_{\bar{\alpha}}(Y^{m_i}, X^1)$ . Similarly for rules with a left to right  $\beta$  edge we establish  $\text{link}_\beta(Z^r, Y^{m_i})$ , while for rules with a right to left  $\beta$  edge we establish  $\text{link}_{\bar{\beta}}(Z^r, Y^{m_i})$ . To ensure that these links can be established if and only if the corresponding GPS rule is authorized we define  $f_\sigma$  from a type  $s$  subject to be as follows.

Class of rule i	GPS System X obtains Z/γ using rule i	SPM Simulation X obtains Z/γc as follows			
		Copy	From	To	Using
I	Given $X \xrightarrow{\alpha} Y \xrightarrow{\beta} Z$ Obtain $X \xrightarrow{\gamma} Z$	$Z/\beta$ $Z^r/\beta$ $Y/\alpha$ $Y^{m_i}/\alpha$	Y Z X Y	$Y^{m_i}$ $Y^{m_i}$ $X^1$ $X^1$	$\text{link}_k(Y, Y^{m_i})$ $\text{link}_\beta(Z, Y^{m_i})$ $\text{link}_k(X, X^1)$ $\text{link}_\alpha(Y, X^1)$
II	Given $X \xrightarrow{\alpha} Y \xleftarrow{\beta} Z$ Obtain $X \xrightarrow{\gamma} Z$	$Y/\beta$ $Y^{m_i}/\beta$ $Y/\alpha$ $Y^{m_i}/\alpha$	Z Y X Y	$Z^r$ $Z^r$ $X^1$ $X^1$	$\text{link}_k(Z, Z^r)$ $\text{link}_\beta(Y, Z^r)$ $\text{link}_k(X, X^1)$ $\text{link}_\alpha(Y, X^1)$
III	Given $X \xleftarrow{\alpha} Y \xrightarrow{\beta} Z$ Obtain $X \xrightarrow{\gamma} Z$	$Z/\beta$ $Z^r/\beta$ $X/\alpha$ $X^1/\alpha$	Y Z Y X	$Y^{m_i}$ $Y^{m_i}$ $Y^{m_i}$ $Y^{m_i}$	$\text{link}_k(Y, Y^{m_i})$ $\text{link}_\beta(Z, Y^{m_i})$ $\text{link}_k(Y, Y^{m_i})$ $\text{link}_\alpha(X, Y^{m_i})$
IV	Given $X \xleftarrow{\alpha} Y \xleftarrow{\beta} Z$ Obtain $X \xrightarrow{\gamma} Z$	$Y/\beta$ $Y^{m_i}/\beta$ $X/\alpha$ $X^1/\alpha$	Z Y Y X	$Z^r$ $Z^r$ $Y^{m_i}$ $Y^{m_i}$	$\text{link}_k(Z, Z^r)$ $\text{link}_\beta(Y, Z^r)$ $\text{link}_k(Y, Y^{m_i})$ $\text{link}_\alpha(X, Y^{m_i})$
All	Common suffix	$Z^\xi/\gamma c$ $Z^\xi/\gamma c$ $Z^\xi/\gamma c$ $Z^\xi/\gamma c$ $Z/\gamma c$	$Z^\xi$ $Z^r$ $Y^{m_i}$ $X^1$ $Z^\xi$	$Z^r$ $Y^{m_i}$ $X^1$ $X$ $X$	$\text{link}_k(Z^\xi, Z^r)$ $\begin{cases} \text{link}_\beta(Z^r, Y^{m_i}) \\ \text{link}_\beta(Z^r, Y^{m_i}) \end{cases}$ $\begin{cases} \text{link}_\alpha(Y^{m_i}, X^1) \\ \text{link}_\alpha(Y^{m_i}, X^1) \end{cases}$ $\text{link}_k(X^1, X)$ $\text{link}_\gamma(Z^\xi, X)$
					Class I, III Class II, IV Class I, II Class III, IV

Table 6: Simulation of GPS Operations in SPM

$$\begin{aligned}
 f_\sigma(s, sl) &= \{sm_i\} \\
 f_\sigma(s, sr) &= \{sm_i\} \\
 f_\sigma(s, sm_i) &= \{sl/c\} \\
 &\{sr/c\}
 \end{aligned}$$

To continue the simulation ticket in  $\text{dom}(X)$ . This is achieved by placing  $Z^\xi$  in  $\text{dom}(X)$ . Recall that  $Z^\xi$  possesses the inverse-cohort link, by placing  $Z^r$  in  $\text{dom}(X)$ . we allow  $Z^r$  to acquire  $Z^\xi/R$ .  $X^1$  provided rule i of the GPS system. The following definitions.

$$\begin{aligned}
 f_\sigma(sr, sm_i) &= \text{if rule i} \\
 f_\delta(sr, sm_i) &= \text{if rule i} \\
 f_\sigma(sm_i, sl) &= \text{if rule i} \\
 f_\delta(sm_i, sl) &= \text{if rule i}
 \end{aligned}$$

The effect of all this so far is to allow X to obtain  $Z/\gamma c$  from  $\text{dom}(X)$ . Now there is  $f_k(sl, s)$  to be  $sg/R$ , X is able to allow X to obtain  $Z/\gamma c$  from  $\text{dom}(X)$  by defining  $f_\gamma(sg, s)$  to be  $s/\gamma c$ .

The construction is now complete and simply construct the initial GPS system as follows. For the SPM cohorts  $X, X^\xi, X^1, X^r, X^s$  below.

$$\begin{aligned}
 \text{dom}(X) &= \{X, X^\xi, X^1, X^r, X^s\} \\
 \text{dom}(X^1) &= X^1 \\
 \text{dom}(X^r) &= X^r \\
 \text{dom}(X^{m_i}) &= X^{m_i} \\
 \text{dom}(X^\xi) &= X^\xi
 \end{aligned}$$

This would suffice since GPS system. In hand GPS can be easily extended to combined authority and information. need to combine the construction. In doing so we would need to show that therefore important to show that be realized by SPM create operation and simply requires suitable definitions authorize creation by letting the That is,

$$cc(s) = \{s\}$$



SPM Simulation  
obtains  $Z/\gamma c$  as follows

To	Using
$Y^{m_i}$	$\text{link}_k(Y, Y^{m_i})$
$Y^{m_i}$	$\text{link}_\beta(Z, Y^{m_i})$
$X^1$	$\text{link}_k(X, X^1)$
$X^1$	$\text{link}_\alpha(Y, X^1)$
$Z^r$	$\text{link}_k(Z, Z^r)$
$Z^r$	$\text{link}_\beta(Y, Z^r)$
$X^1$	$\text{link}_k(X, X^1)$
$X^1$	$\text{link}_\alpha(Y, X^1)$
$Y^{m_i}$	$\text{link}_k(Y, Y^{m_i})$
$Y^{m_i}$	$\text{link}_\beta(Z, Y^{m_i})$
$Y^{m_i}$	$\text{link}_k(Y, Y^{m_i})$
$Y^{m_i}$	$\text{link}_\alpha(X, Y^{m_i})$
$Z^r$	$\text{link}_k(Z, Z^r)$
$Z^r$	$\text{link}_\beta(Y, Z^r)$
$Y^{m_i}$	$\text{link}_k(Y, Y^{m_i})$
$Y^{m_i}$	$\text{link}_\alpha(X, Y^{m_i})$
$Z^r$	$\text{link}_{\hat{k}}(Z^s, Z^r)$
$Y^{m_i}$	$\begin{cases} \text{link}_\beta(Z^r, Y^{m_i}) & \text{Class I, III} \\ \text{link}_\beta(Z^r, Y^{m_i}) & \text{Class II, IV} \end{cases}$
$X^1$	$\begin{cases} \text{link}_\alpha(Y^{m_i}, X^1) & \text{Class I, II} \\ \text{link}_{\hat{\alpha}}(Y^{m_i}, X^1) & \text{Class III, IV} \end{cases}$
$X$	$\text{link}_{\hat{k}}(X^1, X)$
$X$	$\text{link}_\gamma(Z^s, X)$

operations in SPM

$$\begin{aligned} f_\sigma(s, sl) &= \{sm_i/\sigma \mid \text{for every class I or II rule } i \text{ with } \alpha=\sigma\} \\ f_\sigma(s, sr) &= \{sm_i/\sigma \mid \text{for every class II or IV rule } i \text{ with } \beta=\sigma\} \\ f_\sigma(s, sm_i) &= \{sl/\sigma \mid \text{if rule } i \text{ is of class III or IV with } \alpha=\sigma\} \cup \\ &\quad \{sr/\sigma \mid \text{if rule } i \text{ is of class I or III with } \beta=\sigma\} \end{aligned}$$

To continue the simulation of the GPS rules we somehow have to get the  $Z/\gamma c$  ticket in  $\text{dom}(X)$ . This is achieved by the "common suffix" portion of table 6. Recall that  $Z^s$  possesses the tickets  $Z/R$  and  $Z^s/R$ . We connect  $Z^s$  to  $Z^r$  by an inverse-cohort link, by placing  $Z^r/k$  in  $\text{dom}(Z^s)$ . By defining  $f_{\hat{k}}(sg, sr)$  to be  $sg/R$  we allow  $Z^r$  to acquire  $Z^s/R$ . We allow  $Z^s/\gamma c$  to be copied from  $Z^r$  to  $Y^{m_i}$  to  $X^1$  provided rule  $i$  of the GPS system lets  $X$  obtain  $Z/\gamma$ . We achieve this by the following definitions.

$$\begin{aligned} f_\sigma(sr, sm_i) &= \text{if rule } i \text{ is of class I or III with } \beta=\sigma \text{ then } sg/\gamma c \text{ else } \phi \\ f_{\hat{\sigma}}(sr, sm_i) &= \text{if rule } i \text{ is of class II or IV with } \beta=\sigma \text{ then } sg/\gamma c \text{ else } \phi \\ f_\sigma(sm_i, sl) &= \text{if rule } i \text{ is of class I or II with } \alpha=\sigma \text{ then } sg/\gamma c \text{ else } \phi \\ f_{\hat{\sigma}}(sm_i, sl) &= \text{if rule } i \text{ is of class III or IV with } \alpha=\sigma \text{ then } sg/\gamma c \text{ else } \phi \end{aligned}$$

The effect of all this so far is to enable  $X^1$  to acquire  $Z^s/\gamma c$ , if  $X$  can obtain  $Z/\gamma$  in the GPS system. Now there is an inverse-cohort link from  $X^1$  to  $X$ , so by defining  $f_{\hat{k}}(sl, s)$  to be  $sg/R$ ,  $X$  is able to acquire  $Z^s/\gamma$  in our simulation. The final step is to allow  $X$  to obtain  $Z/\gamma c$  from  $\text{dom}(Z^s)$  over this  $\text{link}_\gamma(Z^r, X)$ . This is easily achieved by defining  $f_\gamma(sg, s)$  to be  $s/\gamma c$ .

The construction is now almost complete. We could in fact stop at this point and simply construct the initial state of the SPM system from the initial state of the GPS system as follows. For each subject  $X$  in the GPS system introduce the SPM cohorts  $X, X^s, X^1, X^r, X^{m_1}, \dots, X^{m_n}$  with their initial domains as given below.

$$\begin{aligned} \text{dom}(X) &= \{Y/\sigma c \mid X \xrightarrow{\sigma} Y \text{ in the GPS system}\} \cup \\ &\quad \{X^1, X^r, X^{m_1}, \dots, X^{m_n}\} \times R \\ \text{dom}(X^1) &= X/k \\ \text{dom}(X^r) &= X/k \\ \text{dom}(X^{m_i}) &= X/k, i=1 \dots n \\ \text{dom}(X^s) &= X/R \cup X^s/R \cup X^r/k \end{aligned}$$

This would suffice since GPS systems have no create operation. On the other hand GPS can be easily extended to include create operations. In fact to simulate combined authority and information transfer and theft in the take-grant model, we need to combine the constructions of this Section with those of the previous one. In doing so we would need to allow creation of subjects and their cohorts. It is therefore important to show that the cohorts with appropriate domains can actually be realized by SPM create operations. For the most part this is straightforward and simply requires suitable definition of  $cc$  and the create-rules. It seems proper to authorize creation by letting the type  $s$  subjects create the other types of cohorts. That is,

$$cc(s) = \{sg, sl, sr, sm_1, sm_2, \dots, sm_n\}$$

All other values of  $cc$  are empty. The initial state can then be defined to simply consist of a type  $s$  subject  $X$  for each GPS subject  $X$  with

$$\text{dom}(X) = \{Y/\sigma c \mid X \xrightarrow{\sigma} Y \text{ in the GPS system}\}$$

Tickets relating  $X$  to its cohorts can then be introduced by the create-rules. A minor complication arises from the requirement that  $X^r/k \in \text{dom}(X^s)$ . We can achieve this by copying  $X^r/k$  from  $X$  to  $X^s$ , for which purpose we define  $f_k(s, sg)$  to be  $sr/k$ . With this set up we can easily extend the construction to allow subject creation by placing  $s$  in  $cc$ .

The above discussion results in the following scheme.

**Scheme 6** Grammatical protection systems with  $n$  rules numbered  $1 \dots n$ .

1.  $TS = \{s, sg, sl, sr, sm_1, sm_2, \dots, sm_n\}$ ,  $TO = \phi$   
The type  $s$  subjects simulate GPS subjects. The other subject types respectively have the following roles: generator cohort, left cohort, right cohort, and middle cohort for GPS rules  $1 \dots n$ .
2.  $R = RG \cup \{k : c\}$ , where  $RG$  is the set of rights in the grammatical protection system extended to occur with and without the copy flag and  $k \notin RG$
3.  $cc(s) = \{sg, sl, sr, sm_1, sm_2, \dots, sm_n\}$
4.  $cr_p(s, sl) = \text{child}/R$ ,  $cr_c(s, sl) = \text{parent}/k$   
 $cr_p(s, sm_i) = \text{child}/R$ ,  $cr_c(s, sm_i) = \text{parent}/k$   
 $cr_p(s, sr) = \text{child}/R$ ,  $cr_c(s, sr) = \text{parent}/k$   
 $cr_p(s, sg) = \phi$ ,  $cr_c(s, sg) = \text{child}/R \cup \text{parent}/R$
5.  $\text{link}_k(U, V) \equiv U/k \in \text{dom}(V)$   
 $\text{link}_{\hat{k}}(U, V) \equiv V/k \in \text{dom}(U)$

For every  $\sigma$  which occurs as  $\alpha$ ,  $\beta$  or  $\gamma$  in any GPS rule,

$$\text{link}_\sigma(U, V) \equiv U/\sigma \in \text{dom}(V) \equiv (\forall p \in \sigma) U/p \in \text{dom}(V)$$

$$\text{link}_{\hat{\sigma}}(U, V) \equiv V/\sigma \in \text{dom}(U) \equiv (\forall p \in \sigma) V/p \in \text{dom}(U)$$

The subscripts on these links have the following mnemonic significance:  $k$  for cohort,  $\hat{k}$  for inverse cohort,  $\sigma$  for sigma, and  $\hat{\sigma}$  for inverse sigma.

6. Undefined values of the filter functions are assumed by default to be  $\phi$ .

$$f_k(s, sl) = s/RG$$

$$f_k(s, sm_i) = s/RG$$

$$f_k(s, sr) = s/RG$$

$$f_k(s, sg) = sr/k$$

$$f_{\hat{k}}(sg, sr) = sg/R$$

$$f_{\hat{k}}(sl, s) = sg/R$$

$$f_\sigma(s, sl) = \{sm_i/\sigma \mid \text{for every class I or II rule } i \text{ with } \sigma=\alpha\}$$

$$f_\sigma(s, sr) = \{sm_i/\sigma \mid \text{for every class II or IV rule } i \text{ with } \sigma=\beta\}$$

$$f_\sigma(s, sm_i) = \{sl/\sigma \mid \text{if rule } i \text{ is of class III or IV with } \sigma=\alpha\} \cup \{sr/\sigma \mid \text{if rule } i \text{ is of class I or III with } \sigma=\beta\}$$

$$f_\sigma(sr, sm_i) = \text{if rule } i \text{ is}$$

$$f_{\hat{\sigma}}(sr, sm_i) = \text{if rule } i \text{ is}$$

$$f_\sigma(sm_i, sl) = \text{if rule } i \text{ is}$$

$$f_{\hat{\sigma}}(sm_i, sl) = \text{if rule } i \text{ is}$$

$$f_\sigma(sg, s) = s/\sigma c$$

In the rest of this Section we pr

**Theorem 2** The GPS system of d  
ified initial state are equivalent.

**Proof:** We prove equivalence in tw  
is a GPS state with  $X \xrightarrow{\gamma} Z$  then  
system by which  $Z/\gamma c \in \text{dom}(X)$ . Th  
as shown in table 6. It is apparent f  
the SPM operations in this simulat  
the converse property given below.

$$Z/r : c \in \text{dom}(X) \Rightarrow \text{there exists}$$

By the results of [34] as discussed in  
safety analysis by assuming that ea  
of each of the remaining types. So f  
only copy operations.

We prove the above assertion by  
the SPM system. For the basis case  
trivially from construction of the in  
states derived by less than  $n$  copy o  
copying  $Z/r : c$  to  $X$ , the assertion f  
inspection of the scheme it is evide  
from  $\text{dom}(Z^s)$  over some  $\text{link}_\sigma$  where  
evident that  $Z^s/\sigma$  can be copied to d  
be established only by the create-rule  
type  $sg/\sigma c$  only from subjects of type  
 $Z^s/\sigma c$  was copied to  $X^1$ . There are fo  
copy  $Z^s/\sigma c$  from  $Y^{m_i}$  to  $X^1$  we requ  
is  $r \in \sigma \subseteq \gamma$ . By definition  $\text{link}_\alpha(Y^{m_i}, X$   
the scheme,  $X^1$  can obtain  $Y^{m_i}/\alpha$  onl  
in turn requires  $Y/\alpha \in \text{dom}(X^1)$ . Subj  
only from subjects of type  $s$  over a  
by the create-rules, so  $X^1$  must have  
that  $Y/\alpha c \in \text{dom}(X)$ . Therefore by in  
 $X \xrightarrow{\alpha} Y$ .

Next consider the requirement tha  
can conclude that there is a GPS sta

Since GPS systems are monotonic  
 $X \xrightarrow{\alpha} Y$  and with  $Y \xrightarrow{\beta} Z$  then th

ate can then be defined to simply  
ect X with

the GPS system}

duced by the create-rules. A minor  
r/k ∈ dom(X<sup>g</sup>). We can achieve this  
oose we define f<sub>k</sub>(s, sg) to be sr/k.  
struction to allow subject creation

scheme.

th n rules numbered 1 ... n.

CO = φ

s. The other subject types respec-  
r cohort, left cohort, right cohort,

ights in the grammatical protection  
ut the copy flag and k ∉ RG

= parent/k  
= parent/k  
= parent/k  
= child/R ∪ parent/R

any GPS rule,

U/p ∈ dom(V)  
U/p ∈ dom(U)

lowing mnemonic significance: k for  
and σ̂ for inverse sigma.

re assumed by default to be φ.

ss I or II rule i with σ=α}  
ss II or IV rule i with σ=β}  
ss III or IV with σ=α} ∪  
ss I or III with σ=β}

$$\begin{aligned} f_{\sigma}(sr, sm_i) &= \text{if rule } i \text{ is of class I or III with } \sigma=\beta \text{ then } sg/\gamma c \text{ else } \phi \\ f_{\hat{\sigma}}(sr, sm_i) &= \text{if rule } i \text{ is of class II or IV with } \sigma=\beta \text{ then } sg/\gamma c \text{ else } \phi \\ f_{\sigma}(sm_i, sl) &= \text{if rule } i \text{ is of class I or II with } \sigma=\alpha \text{ then } sg/\gamma c \text{ else } \phi \\ f_{\hat{\sigma}}(sm_i, sl) &= \text{if rule } i \text{ is of class III or IV with } \sigma=\alpha \text{ then } sg/\gamma c \text{ else } \phi \\ f_{\sigma}(sg, s) &= s/\sigma c \end{aligned}$$

In the rest of this Section we prove the correctness of our construction.

**Theorem 2** *The GPS system of definition 7 and the SPM scheme 6 with its specified initial state are equivalent.*

**Proof:** We prove equivalence in two steps. Firstly we need to show that if there is a GPS state with  $X \xrightarrow{\gamma} Z$  then there is a sequence of operations in the SPM system by which  $Z/\gamma c \in \text{dom}(X)$ . This is achieved by simulating each GPS operation as shown in table 6. It is apparent from our discussion leading up to scheme 6 that the SPM operations in this simulation are authorized. Secondly we need to show the converse property given below.

$Z/r : c \in \text{dom}(X) \Rightarrow$  there exists a GPS state in which  $X \xrightarrow{\gamma} Z$  with  $r \in \gamma$

By the results of [34] as discussed in Section 3, we can ignore create operations for safety analysis by assuming that each SPM subject of type  $s$  creates one instance of each of the remaining types. So from this augmented state we need to consider only copy operations.

We prove the above assertion by induction on the number of copy operations in the SPM system. For the basis case let this number be 0 and the assertion follows trivially from construction of the initial state. Assume the assertion is true for states derived by less than  $n$  copy operations. If the  $n$ -th operation is other than copying  $Z/r : c$  to  $X$ , the assertion follows by induction hypothesis. Otherwise by inspection of the scheme it is evident that  $Z/r : c$  can be copied to  $\text{dom}(X)$  only from  $\text{dom}(Z^g)$  over some  $\text{link}_{\sigma}$  where  $r \in \sigma$ . This requires  $Z^g/\sigma \in \text{dom}(X)$ . It is further evident that  $Z^g/\sigma$  can be copied to  $\text{dom}(X)$  only from  $\text{dom}(X^1)$  over  $\text{link}_k$ , which can be established only by the create-rules. Now subjects of type  $sl$  can obtain tickets of type  $sg/\sigma c$  only from subjects of type  $sm_i$ . So there must exist some  $Y^{mi}$  from which  $Z^g/\sigma c$  was copied to  $X^1$ . There are four cases to consider. Let rule  $i$  be of class I. To copy  $Z^g/\sigma c$  from  $Y^{mi}$  to  $X^1$  we require  $\text{link}_{\alpha}(Y^{mi}, X^1)$  and  $sg/\sigma \in f_{\alpha}(sm_i, sl)$ . That is  $r \in \sigma \subseteq \gamma$ . By definition  $\text{link}_{\alpha}(Y^{mi}, X^1)$  implies  $Y^{mi}/\alpha \in \text{dom}(X^1)$ . By inspection of the scheme,  $X^1$  can obtain  $Y^{mi}/\alpha$  only by copying it from  $Y$  over  $\text{link}_{\alpha}(Y, X^1)$ . This in turn requires  $Y/\alpha \in \text{dom}(X^1)$ . Subjects of type  $sl$  can obtain tickets of type  $s/\alpha$  only from subjects of type  $s$  over a  $\text{link}_k$ . Such  $\text{link}_k$ 's can be established only by the create-rules, so  $X^1$  must have obtained  $Y/\alpha$  from  $\text{dom}(X)$ , which requires that  $Y/\alpha c \in \text{dom}(X)$ . Therefore by induction hypothesis there is a GPS state with  $X \xrightarrow{\alpha} Y$ .

Next consider the requirement that  $Z^g/\sigma c \in \text{dom}(Y^{mi})$ . By similar arguments we can conclude that there is a GPS state with  $Y \xrightarrow{\beta} Z$ .

Since GPS systems are monotonic it follows that if there are GPS states with  $X \xrightarrow{\alpha} Y$  and with  $Y \xrightarrow{\beta} Z$  then there is a GPS state with  $X \xrightarrow{\alpha} Y \xrightarrow{\beta} Z$ . In

this GPS state rule  $i$  is authorized so  $X$  can obtain  $\gamma$  rights for  $Z$ , where  $r \in \gamma$ . This completes the induction step when rule  $i$  is of class I. For the other cases where rule  $i$  is of class II, III or IV the induction step can be similarly proved.  $\square$

## 8 Conclusion

In this paper we have shown how versions of Bell-LaPadula multilevel security model [3], take-grant models [16, 21, 41] and grammatical protection systems [10, 22] can be specified as SPM schemes. This work complements our earlier efforts in demonstrating the modeling power of SPM by considering specific policies of practical interest [32, 33, 34, 36]. It is encouraging that SPM offers a unified framework in which these diverse models and policies can be expressed. It is moreover remarkable, that in all these cases the SPM schemes satisfy the acyclic attenuating assumption required for safety analysis [34].

The results of this paper are in sharp contrast to results for the Harrison-Ruzzo-Ullman (HRU) access-matrix model [14]. HRU does subsume all the models discussed in this paper in terms of expressive power. However, all known constructions of these models within HRU require multi-conditional commands (i.e., commands whose conditions have two or more terms), whereas safety is undecidable in HRU even for bi-conditional commands (i.e., commands whose conditions have exactly two terms).

Our construction for multilevel models establishes that the traditional label-based mandatory controls of multilevel security have an alternate expression in terms of the type-based constraints on propagation of access rights imposed by SPM. The SPM viewpoint has the advantage of providing explicit machinery for formulating policies "in between" the two extremes of mandatory and discretionary policies in the Bell-LaPadula model.

The construction for theft in take-grant models emphasizes that a protection model with the generality of SPM is useful, even for a system with very specific control operations. This is because assumptions about behavior can be modeled in SPM. Most systems implement very specific control operations, and safety can be guaranteed only with such additional assumptions.

Our construction for grammatical protection systems demonstrates the ability of SPM to simulate models whose control operations appear to be contrary to SPM operations. GPS rules are particularly troublesome in this regard, since they actually allow new privileges to be created. This indicates that SPM has abstracted some essential properties of control operations in protection models. This abstraction is probably more fundamental than the viewpoint which lead us to develop the SPM rules in the first place. The GPS construction is also significant, because in combination with the take-grant constructions it allows us to accommodate the take-grant notions of information and authority transfer and theft within SPM.

We conjecture that SPM is in some sense equivalent to the monotonic access matrix, in which delete and destroy operations are not allowed [15]. Some kind of equivalence is inevitable, since both models have undecidable safety in general and both are monotonic. The interesting question is whether or not SPM has

behavioral equivalence to monotonic Resolution of this question will provide of protection models. It has recently been extending SPM to have a multi-parent monotonic HRU. It has also been compared than monotonic HRU (under the terminology relationship of the expressive power of monotonic HRU remains an important

Finally, we are well aware that SPM extending it to include some non-monotonic and mutually exclusive privileges is a challenge. Some aspects of the integrity policies and others [29, 35, 42] will need such features. It does not take very much to get into the monotonic privileges. Developing a system with privileges and has tractable safety analysis is a problem. Our work on SPM provides

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obtain  $\gamma$  rights for  $Z$ , where  $r \in \gamma$ . This is of class I. For the other cases where step can be similarly proved.  $\square$

ns of Bell-LaPadula multilevel security and grammatical protection systems [10], work complements our earlier efforts SPM by considering specific policies of encouraging that SPM offers a unified and policies can be expressed. It is s the SPM schemes satisfy the acyclic analysis [34].

trast to results for the Harrison-Ruzzo-HRU does subsume all the models dis- power. However, all known constructions conditional commands (i.e., commands whereas safety is undecidable in HRU commands whose conditions have exactly

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ction systems demonstrates the ability operations appear to be contrary to arly troublesome in this regard, since eated. This indicates that SPM has ntrol operations in protection models. ental than the viewpoint which lead lace. The GPS construction is also e take-grant constructions it allows us nformation and authority transfer and

se equivalent to the monotonic access ions are not allowed [15]. Some kind dels have undecidable safety in general question is whether or not SPM has

behavioral equivalence to monotonic HRU, in the sense discussed in Section 4. Resolution of this question will provide a significant advance in our understanding of protection models. It has recently been shown by Ammann and Sandhu [1, 2] that extending SPM to have a multi-parent joint create operation gives us equivalence to monotonic HRU. It has also been conjectured that SPM is actually less expressive than monotonic HRU (under the terms of behavioral equivalence). The precise relationship of the expressive power of SPM with respect to extended SPM or monotonic HRU remains an important open question.

Finally, we are well aware that SPM is a monotonic model. The question of extending it to include some non-monotonic features such as transfer-only privileges and mutually exclusive privileges is an important research issue. It appears that some aspects of the integrity policies considered by Clark and Wilson [11] and others [29, 35, 42] will need such features. However as demonstrated by Budd [10] it does not take very much to get into intractable analysis problems with such non-monotonic privileges. Developing a suitable model which includes non-monotonic privileges and has tractable safety analysis is an important and difficult research problem. Our work on SPM provides a basis for this research.

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## A LOGICAL VIEW

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### Abstract

In the context of the modal logic  $K_B\varphi \rightarrow R_B\varphi$  that could be read "if a subject knows  $\varphi$  then he knows that he knows  $\varphi$ ". We propose a new definition of security (in terms of dependencies) between objects. We study the dependencies of security with non-interference, especially with respect to

### 1 Introduction

Highest ratings of evaluation are given to systems with respect to its security specifications. Finding such ratings, finding a formal model for such ratings, still an open subject of research. Several directions followed several directions.

The main direction is to model security as state machines and absence of information. Security is defined as a constraint on the state transitions. Various definitions of absence of information are proposed by Goguen and McCullough in [McC87].

Another direction for formalizing security is modal logics. The work by Goguen and McCullough results (see [GMP90]). The work by Goguen (ity) as a very simple formula

"If B knows that

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